

# Response to ”Comment on: ‘Thermodynamics of viscoelastic fluids: the temperature equation’ ”

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The thermodynamics of irreversible processes is based on the principle of local action, i.e. the principle of local and instantaneous equilibrium. This assumption means that, although the system as a whole is not in equilibrium, the material can be divided into small physical subelements, where each subelement forms a small homogeneous equilibrium system. All basic assumptions, including this one, in the theory of internal variables are discussed in detail by Kuiken (1994) (Chapter 3). We will only summarise his comments on the principle of local action in relation to the temperature here. Measurement of the temperature requires that an equilibrium situation between thermometer and fluid exists. In nonequilibrium situations, measurement of the temperature is only meaningful if the thermometer is at least surrounded by a small amount of fluid that is instantaneously in equilibrium with the thermometer. Here small means small compared to dimensions over which the temperature varies measurably, and large compared to the thermometer. Instantaneously means that the time scale of the temperature change in the fluid is large compared to the time scale of the thermometer. The principle of local action ensures that equilibrium quantities like temperature, pressure, and entropy are well defined in nonequilibrium situations. On the other hand, this principle, of course, restricts the area of application of the thermodynamic theory somewhat. If this is a serious disadvantage needs to be established by experiments. The principle of local equilibrium is not only needed for the temperature equation, but is a main assumption in the thermodynamic theory. It is, for example, also used by Jongschaap (1994) and Leonov (1987) to derive the (isothermal) viscoelastic constitutive equations from the thermodynamics. As we describe the influences of the viscoelasticity on the temperature equation, it seems plausible that our temperature equation is valid whenever the stress constitutive equations are. For this, the discussion on the temperature and its measurement is only meaningful for ‘real non-equilibrium’ situations, i.e. when there is no local and instantaneous equilibrium, and not for the theory in our paper.

In the penultimate paragraph Prof. Grmela states that we suggest another setting for non-isothermal rheology. This is not exactly the case: in our article we have only included one type of internal variable, the configuration tensor. This quantity describes the viscoelasticity, and all well-known differential models can be obtained with it. Our purpose was to demonstrate which effects the viscoelasticity (by means of the configuration tensor) has on the temperature equation, i.e. how does the temperature equation change when the configuration tensor is introduced, and to relate the obtained effects to existing experimental data. Our purpose was not to give some general setting for nonisothermal flow. For this, we have excluded possible scalar and vectorial relaxation processes. In the section ‘Thermodynamics of viscoelastic fluids: general’.

Including those ones would not be very relevant, because there are no experimental data for validation. Until experimental results become available, it thus remains an open question whether other relaxation processes play a role, and whether our temperature equation (and possibly also the stress equation) thus has to be extended with other internal variables. Possible options are a vector internal variable for the heat flux in case of large temperature gradients and a scalar internal variable for volume relaxation in case of high pressure gradients (see Kuiken (1994) for details). However, as we do not intend to give a more general setting than the configuration tensor, comparing the settings of our paper and of Grmela is therefore not very meaningful.

The fact that we do not have relaxation of the heat flux due to the temperature gradient (the Cattaneo equation) is directly caused by not taking into account vectorial internal variables, describing the relaxation of the heat flux. For the details of the derivation see Kuiken (1994). Note that by relating the anisotropic heat conduction to the configuration tensor describing the polymer chains, we also have relaxation of the heat flux, but due to the dependence of the heat conduction tensor on the configuration tensor (in a similar manner as the elastic stress has relaxation). As far as we know, no experimental data are available that indicate that the Cattaneo equation is important for viscoelastic fluids, or that the effect is relevant compared to the relaxation caused by the anisotropic heat conduction in the polymer chains.

Summarising, we think that Prof. Grmela's Letter to the Editor is not very relevant in relation to our paper. The temperature seems a problem in his thermodynamics and not for ours. Furthermore, our settings were restricted by the fact that we only wanted to show the effect of viscoelasticity on the temperature equation. Not to include possible internal variables for which no experimental evidence exists. Contrarily to Prof. Grmela, we would turn things around: a temperature equation for viscoelastic fluids, should minimally contain the nonisothermal effects described in our temperature equation. If necessary it can be extended.

## References

- [1] R.J.J. Jongschaap, K.H. de Haas and C.A.J. Damen, *A generic matrix representation of configuration tensor rheological models*, J. Rheol. **38**, 769-796 (1994).
- [2] G.D.C. Kuiken, *Thermodynamics of irreversible processes*, Wiley, Chichester, UK, 1994.
- [3] A.I. Leonov, *On a class of constitutive equations for viscoelastic fluids*, J. Non-Newtonian Fluid Mech. **25**, 1-59 (1987).