# Fiber Orientation Kinetics of a Concentrated Short Glass Fiber Suspension in Start up of Shear Flow

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# Abstract

The common approach for simulating the evolution of fiber orientation during flow in concentrated suspensions is to use an empirically modified form of Jeffery's equation referred to as the Folgar-Tucker model. Direct measurements of fiber orientation were performed in startup of shear flow for a 30 wt % short glass fiber-filled polybutylene terephthalate (PBT-30); a matrix that behaves similar to a Newtonian fluid. The experimental results show that the fiber orientation evolves much slower than the models predict. Furthermore, fiber orientation measurements suggest that a steady state is not reached as the fiber orientation continues to slowly evolve, even up to 200 strain units. This is in contrast to the model predictions that predict a steady state is reached in roughly 50 strain units.

# Introduction

Current approaches to predicting the rheology and microstructure of concentrated suspensions use an empirical modification form of Jeffery's equation, the Folgar-Tucker (F-T) model [1]. Previous attempts by researchers to improve on theoretical predictions for non-dilute suspensions have focused on the equations governing the extra stress contributions [2]. Conversely, the equation describing the evolution of the fiber orientation has remained relatively unchanged. The goal of this paper is to assess the ability of the Folgar-Tucker model to accurately describe the evolution of the glass fiber orientation in concentrated suspensions typical of composite fluids.

#### Theory

A compact way to describe the average orientation is with the orientation order parameter tensor, **A**, which is defined as the second moment of the orientation distribution function:

$$\mathbf{A}(t) = \int \mathbf{u} \mathbf{u} \, \psi(\mathbf{u}, t) d\mathbf{u} \tag{1}$$

The trace of **A** is always equal to 1 and for a completely random orientation  $\mathbf{A} = 1/3 \mathbf{I}$ , where **I** is the identity tensor. In the limit where all the fibers are perfectly aligned in the  $x_1$  the only non-zero component is  $A_{11} = 1$ .

For simple flows the F-T model can be written in terms of  $\mathbf{A}$  as follows with the use of the quadratic closure approximation:

$$\frac{d\mathbf{A}}{dt} = \left(\mathbf{A} \cdot \mathbf{W} - \mathbf{W} \cdot \mathbf{A}\right) + \dots$$
(2)  
$$\dots \lambda \left(\mathbf{D} \cdot \mathbf{A} + \mathbf{A} \cdot \mathbf{D} - \mathbf{D} : \mathbf{A}\mathbf{A}\right) + 2C_1 \left|\mathbf{D}\right| \left(\mathbf{I} - 3\mathbf{A}\right)$$

where  $\mathbf{W} = [(\nabla v) - (\nabla v)^t]/2$ ,  $\mathbf{D} = [(\nabla v) + (\nabla v)^t]/2$ ,  $\nabla v$  is the velocity gradient and  $\lambda$  is a constant defining the ellipticity of the particle. For fibers it is common to assume the particle's aspect ratio approaches infinity and to use  $\lambda = 1$ . In the results and discussion section the predictions of Eq. (2) with  $\lambda = 1$  for simple shear flow kinematics are compared to experimental results. The equations were solved numerically for simple shear flow kinematics ( $v_1 = \gamma^* y$  and  $v_2 = v_3 = 0$ ) with  $\lambda = 1$ . Gears implicit predictor-corrector method for stiff differential equations was used to solve the coupled equations using a time step of 0.01 s. The initial conditions were found experimentally.

### **Experimental**

All experiments were performed on a 30 wt% (17.6 vol%) short glass fiber-filled polybutylene terephthalate (PBT-30). The neat PBT suspending medium exhibited little shear thinning behavior shown by way of the magnitude of the complex viscosity,  $|\eta^*| : |\eta^*| = 420$  Pa\*s at 0.1 rad/s,  $|\eta^*| = 320$  Pa\*s at 100 rad/s. Hence, the suspending medium behaved similarly to a Newtonian fluid. The glass fiber has a number average length of  $L_n = 0.36$ mm and aspect ratio  $a_r \sim 30$ . All rheological measurements were performed on a Rheometrics Mechanical Spectrometer (RMS-800) fitted with 50

mm diameter cone and plate geometry to ensure a homogeneous shear field within the rheometer gap. Samples were pre-formed and a 25.4 mm diameter hole was drilled through the center creating a "donut" shaped sample. This ensured that at every position the gap was always greater than two times the number average fiber length. All samples were dried at 120 °C for a minimum of four hours in a vacuum oven at a pressure of 0.3 (in.Hg) before sample molding or testing.

To characterize the evolving fiber orientation under dynamic conditions, donut samples composed of PBT-30 were deformed using the RMS-800 with the cone and plate geometry at a shear rate of  $1 \text{ s}^{-1}$  for a specified amount of time (strain) at 260 °C in a nitrogen environment. Directly after deformation the sample temperature was lowered below the suspension melt temperature, "freezing" the flow induced fiber orientation. The samples were then polished a specific planes and images of the polished surfaces were taken using a Zeiss LSM510 confocal laser scanning microscope fitted with a 40x water immersion objective lens and a laser excitation wavelength of 543 nm. The fiber orientation was characterized using a similar procedure to that described by [3].

#### **Results and Discussion**

The experimental fiber orientation and model predictions can be seen in Fig. 1. The fiber orientation begins from a relatively planar orientation state with most of the fibers oriented in the flow,  $x_1$ , and neutral directions,  $x_3$ , and few fibers oriented in the direction of velocity gradient,  $x_2$ . Upon inception of flow, the fibers evolve to align themselves in the flow direction. Though the fiber orientation reached a statistical steady state by 100 strain units, the average appears to be continuously evolving even up to 200 strain units. The experimentally determined Aii components vs. strain are compared to the F-T model with small  $C_{\rm I}$  ( $C_{\rm I} = 0.0001$ ) and the F-T model with  $C_{\rm I} = 0.006$ , which was determined by fitting the steady state model predictions to the experimental data at a strain of 200. The F-T model with small  $C_{\rm I}$ drastically over predicts the Aii components rate of reorientation. The F-T model with  $C_{\rm I} = 0.006$  also over predicts the Aii components reorientation rate at small strains but exhibits a similar value at large strains as a result of the fit  $C_{\rm I}$ . Furthermore, the F-T model with  $C_{\rm I} = 0.006$  reaches a steady state in ~ 50 strain units. The predictions of the A<sub>ii</sub> components using the F-T model shown in Fig. 1 are relatively accurate at the largest strain which is expected as the  $C_{\rm I}$  was parameter was adjust to fit the F-T model predictions at that strain. However, the continuously

evolving fiber orientation suggests that a correct value would be smaller than  $C_{\rm I} = 0.006$  or some function of the fiber orientation.



**Figure 1.** Experimental and predicted fiber orientation represented through the A<sub>ii</sub> components in startup of simple shear flow at  $\dot{\gamma} = 1 \text{ s}^{-1}$ . The lines represent the predictions of the F-T model with  $C_{\rm I} = 0.0001$  (dashed line) and  $C_{\rm I} = 0.006$  (solid line).

# Conclusions

Experimentally determined fiber orientation was compared to predictions of the F-T model for evolving fiber orientation in start up of simple shear flow. It was found that the model over predicts the rate of fiber reorientation. Furthermore, changing the  $C_{\rm I}$  parameter the model was able to fit the steady state fiber orientation, but the experimental fiber orientation appears to not reach a steady state even up to 200 strain units.

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