Common Laplace transforms

1. $\mathcal{L}\{h(t)\} = \dfrac{1}{s}$

14. $\mathcal{L}\{h(t-\alpha)\} = \dfrac{\mathrm{e}^{-\alpha s}}{s}$

2. $\mathcal{L}\{t^n\} = \dfrac{n!}{s^{n+1}} \qquad n = 0, 1, \ldots$

3. $\mathcal{L}\{\mathrm{e}^{\alpha t}\} = \dfrac{1}{s-\alpha}$

4. $\mathcal{L}\{\sin(\omega t)\} = \dfrac{\omega}{s^2+\omega^2}$

5. $\mathcal{L}\{\cos(\omega t)\} = \dfrac{s}{s^2+\omega^2}$

6. $\mathcal{L}\{\sinh(\omega t)\} = \dfrac{\omega}{s^2-\omega^2}$

7. $\mathcal{L}\{\cosh(\omega t)\} = \dfrac{s}{s^2-\omega^2}$

Five operational properties for Laplace transforms $F(s) = \mathcal{L}\{f(t)\}$

(A) Linearity: $\mathcal{L}\{c_1 f_1(t) + c_2 f_2(t)\} = c_1\mathcal{L}\{f_1(t)\} + c_2\mathcal{L}\{f_2(t)\}$

(B) First shift theorem: $\mathcal{L}\{\mathrm{e}^{\alpha t} f(t)\} = F(s-\alpha)$

(C) Second shift theorem: $\mathcal{L}\{f(t-\alpha)h(t-\alpha)\} = \mathrm{e}^{-\alpha s}F(s)$

(D) Antiderivative: $\mathcal{L}\left\{\displaystyle\int_0^t f(\tau)\,\mathrm{d}\tau\right\} = \dfrac{F(s)}{s}$

(E) n-th derivative: $\mathcal{L}\{f^{(n)}(t)\} = s^n F(s) - s^{n-1} f(0) - \cdots - f^{(n-1)}(0) \qquad n = 1, 2, \ldots$

- $n = 1$: $\mathcal{L}\{f'(t)\} = s^1 F(s) - s^0 f(0) = sF(s) - f(0)$
- $n = 2$: $\mathcal{L}\{f''(t)\} = s^2 F(s) - s^1 f(0) - s^0 f'(0) = s^2F(s) - sf(0) - f'(0)$

Common Integrals

1. $\displaystyle\int \mathrm{e}^{au}\sin(bu)\,\mathrm{d}u = \frac{\mathrm{e}^{au}[a\sin(bu) - b\cos(bu)]}{a^2+b^2} + C$

2. $\displaystyle\int \mathrm{e}^{au}\cos(bu)\,\mathrm{d}u = \frac{\mathrm{e}^{au}[a\cos(bu) + b\sin(bu)]}{a^2+b^2} + C$

3. $\displaystyle\int \sin(au)\sin(bu)\,\mathrm{d}u = -\frac{\sin[(a+b)u]}{2(a+b)} + \frac{\sin[(a-b)u]}{2(a-b)} + C \qquad \underline{a^2 \neq b^2}$

4. $\displaystyle\int \sin(au)\cos(bu)\,\mathrm{d}u = -\frac{\cos[(a+b)u]}{2(a+b)} - \frac{\cos[(a-b)u]}{2(a-b)} + C \qquad \underline{a^2 \neq b^2}$

5. $\displaystyle\int \cos(au)\cos(bu)\,\mathrm{d}u = \frac{\sin[(a+b)u]}{2(a+b)} + \frac{\sin[(a-b)u]}{2(a-b)} + C \qquad \underline{a^2 \neq b^2}$