## **14.3:** Partial Derivatives

■ Notation for regular derivatives of f = f(x)

1. 
$$\frac{\mathrm{d}f}{\mathrm{d}x}$$
 Straight d's

2. f'

#### • Notation for partial derivatives of f = f(x, y, z)

$$egin{array}{lll} 1. \ \displaystylerac{\partial f}{\partial x} \,, \ \displaystylerac{\partial f}{\partial y} \,, \ \displaystylerac{\partial f}{\partial z} & {
m Round} \ \partial {
m 's} \ \end{array} \ 2. \ \displaystylef_x \,, \ \displaystylef_y \,, \ \displaystylef_z \end{array}$$

• Notation for partial derivatives of  $f = f(x_1, \ldots, x_n)$ 

 $rac{\partial f}{\partial x_1}\,,\,\,rac{\partial f}{\partial x_2}\,,\,\cdots,\,\,rac{\partial f}{\partial x_n}$ 

### **14.3:** Partial Derivatives

**Notation for second partial derivatives of** f = f(x, y)

$$f_{xx} = \frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial x} \right) \qquad \qquad f_{xy} = (f_x)_y = \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial x} \right) = \frac{\partial^2 f}{\partial y \partial x}$$
$$f_{yy} = \frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial y} \right) \qquad \qquad f_{yx} = (f_y)_x = \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial y} \right) = \frac{\partial^2 f}{\partial x \partial y}$$

Notation for second partial derivatives of  $f = f(x_1, \ldots, x_n)$ 

$$rac{\partial^2 f}{\partial x_1^2}\,,\,\,rac{\partial^2 f}{\partial x_1\partial x_2}\,,\,\cdots,\,\,rac{\partial^2 f}{\partial x_1\partial x_n} ext{ etc.}$$

Notation for third partial derivatives of f = f(x, y, z)

$$f_{xyz} \;=\; rac{\partial^3 f}{\partial z \partial y \partial x} \;=\; rac{\partial}{\partial z} \left[ rac{\partial}{\partial y} \left( rac{\partial f}{\partial x} 
ight) 
ight] \, ext{etc.}$$

# **14.3:** Partial Derivatives

#### • Computing second partial derivatives

• Example: Compute second partial derivatives of  $f(x,y) = x^2 + xy$ 

| $f_x=2x+y$ | $(y  { m constant})$  |
|------------|-----------------------|
| $f_y=0+x$  | $(x 	ext{ constant})$ |

$$egin{aligned} f_{xx} &= rac{\partial}{\partial x} \left( f_x 
ight) = rac{\partial}{\partial x} \Big( 2x + y \Big) = 2 & (y ext{ constant}) \ f_{xy} &= rac{\partial}{\partial y} \left( f_x 
ight) = rac{\partial}{\partial y} \Big( 2x + y \Big) = 1 & (x ext{ constant}) \ f_{yx} &= rac{\partial}{\partial x} \left( f_y 
ight) = rac{\partial}{\partial x} \Big( x \Big) = 1 & (y ext{ constant}) \ f_{yy} &= rac{\partial}{\partial y} \left( f_y 
ight) = rac{\partial}{\partial y} \Big( x \Big) = 0 & (x ext{ constant}) \end{aligned}$$