8.3: Gradient Fields

■ Examples of gradient fields

- **Electric field**: $\underline{\boldsymbol{E}} = -\underline{\boldsymbol{\nabla}}\boldsymbol{V}$

  $\boldsymbol{V}$: **electric potential**

- **Gravitational field**: $\underline{\boldsymbol{F}} = -\underline{\boldsymbol{\nabla}}\boldsymbol{V}$

  $\boldsymbol{V}$: **gravitational potential energy**

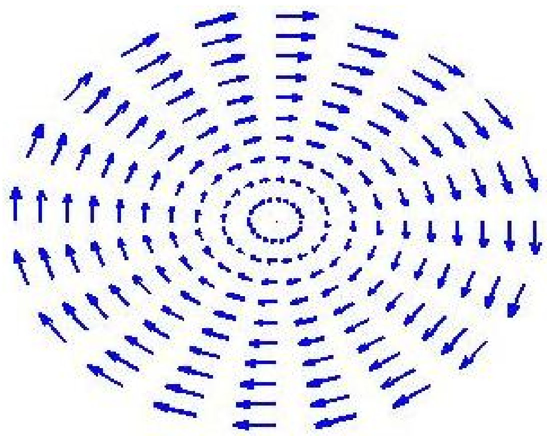
- **Velocity field**: $\underline{\boldsymbol{v}} = -\underline{\boldsymbol{\nabla}}\boldsymbol{\phi}$

  $\boldsymbol{\phi}$: **velocity potential (used in irrotational flows)**

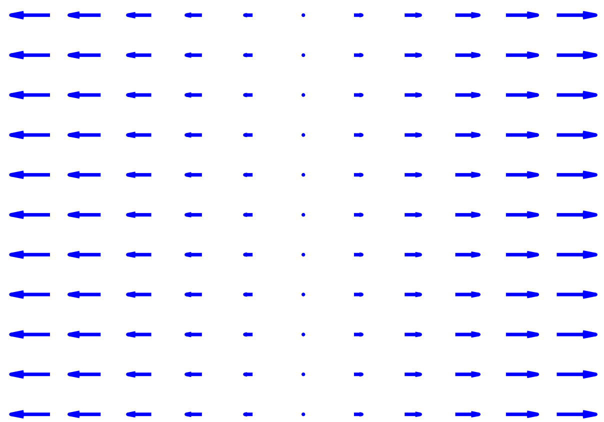
8.4/4.4: Interpretation of $\underline{\nabla} \cdot \underline{F}$ in $\mathbb{R}^2$

Is $\underline{\nabla} \cdot \underline{F}$ positive, negative, or zero?

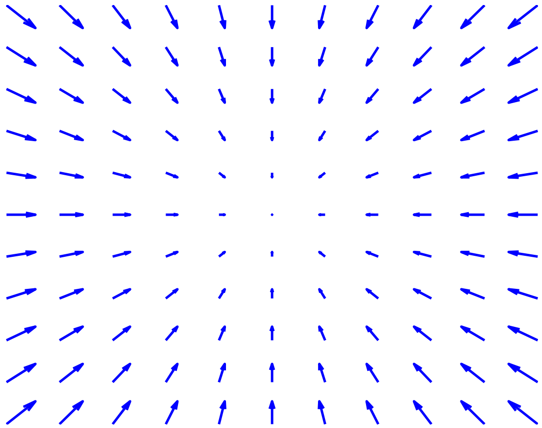
A



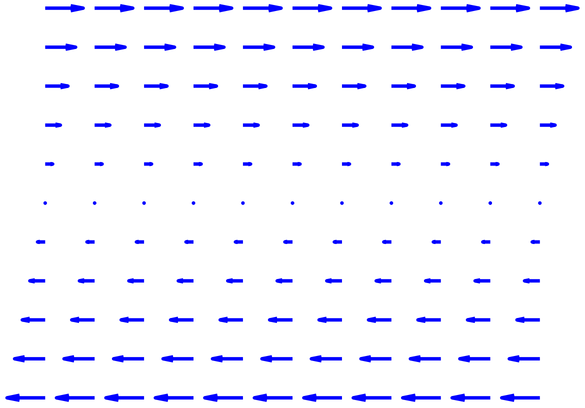
B



C



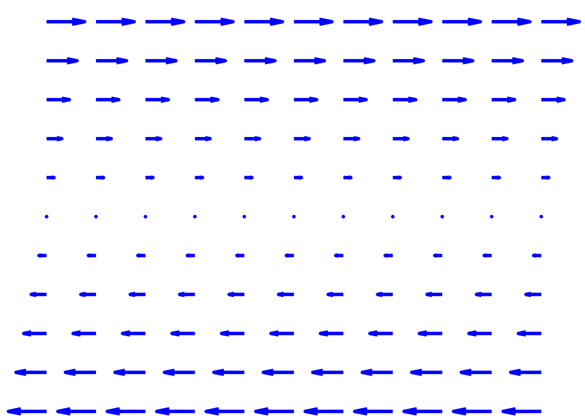
D



Sec. 8.2/4.4: Interpretation of $(\underline{\boldsymbol{\nabla}} \times \underline{\boldsymbol{F}}) \cdot \underline{\boldsymbol{k}}$ in $\mathbb{R}^2$

Is $K(x, y) = (\underline{\boldsymbol{\nabla}} \times \underline{\boldsymbol{F}}) \cdot \underline{\boldsymbol{k}}$ positive, negative, or zero?

A



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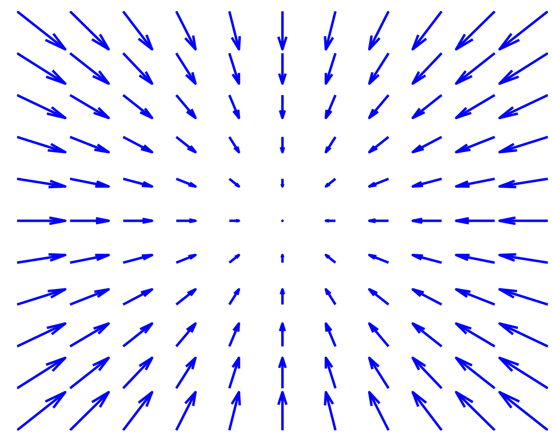
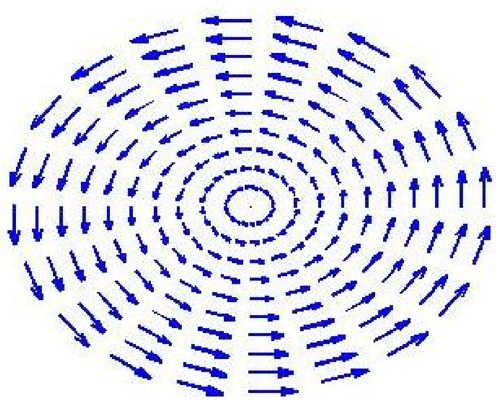
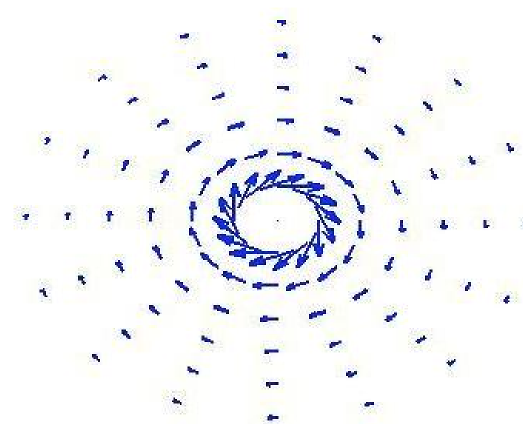


C



D



Choosing Integral Theorems or Directly

1. Compute $\int_{\underline{c}} \underline{F} \cdot \, \mathrm{d}\underline{s}$ where $\underline{c}(t) = (3\cos t,\ 3\sin t,\ t)$ with $0 \leq t \leq \pi$ and $\underline{F}(x, y, z) = (\mathrm{e}^y \cos z,\ x\mathrm{e}^y \cos z,\ -xe^y \sin z)$.

2. Compute the flow rate through the surface $x^2 + y^2 + z^2 = 1$ with $z \geq 0$.

   The surface is oriented by the normal pointing away from the origin.

   Velocity: $\underline{v}(x, y, z) = (x,\ y,\ z)$.

3. Compute $\iint_S \underline{\nabla} \cdot (x + y,\ x - y,\ z^2)\, \mathrm{d}S$ where S is the unit sphere.

4. Compute $\iint_S (\underline{\nabla} \times \underline{F}) \cdot \, \mathrm{d}\underline{S}$ with $\underline{F}(x, y, z) = \left(\sin y^3,\ xy^2z^3,\ \dfrac{1}{1 + x^2y^2z^2}\right)$.

   S: union of $z = 2 - x^2 - y^2$ with $1 \leq z \leq 2$ and $z = x^2 + y^2$ with $0 \leq z \leq 1$.

Choosing Integral Theorems or Directly

5. Compute $\iint_{\partial W} z^2 \, \mathrm{d}S$ where W is the unit cube.

6. Compute $\int_{\underline{c}} (\mathrm{e}^{1/(x^2+1)}, \; z, \; y) \cdot \, \mathrm{d}\underline{s}$

   $\underline{c}$ is the boundary of $z = x^2 + y^2$ with $0 \le z \le 2$.

   $\underline{c}$ is oriented clockwise when viewed from the top.

7. Compute $\int_{\underline{c}} (x+y) \, \mathrm{d}s$ along the closed curve $\underline{c}(t) = (\cos t, \; \sin t)$ with $0 \le t \le 2\pi$.

8. Compute $\int_{\underline{c}} x^7 \, \mathrm{d}x + \mathrm{e}^{y^4} \, \mathrm{d}y$ where $\underline{c}(t) = (3\cos t, \; \sin t)$ with $0 \le t \le 2\pi$.

9. Compute the flow rate out of $x^2 + y^2 + z^2 = 4$. Velocity: $\underline{v}(x, y, z) = (3x, \; z, \; x)$.