Test 3: All Unit 3 notes and posted Unit 3 slides

Basic knowledge

- Integration of basic functions ($\sin t$, $\cos t$, e^t, t^n, $\sqrt{t^n}$); u-substitution; Recognize hard integrals; Dot product; Cross product; Curve and surface parametrization; Partial derivatives; Gradient vector; Curl; Divergence; Set up and evaluation of double integrals in rectangular and polar coordinates; Set up and evaluation of triple integrals in rectangular, cylindrical, and spherical coordinates.

Integration over paths and surfaces; Integral theorems

- **4.2/7.1** Path integral of scalar functions $\int_{\boldsymbol{c}} f \ \mathrm{d}s = \int_a^b f(\boldsymbol{c}(t)) \ ||\boldsymbol{c}'(t)|| \ \mathrm{d}t$

  Arc length; Area of a fence; Mass of a wire; Integration over piecewise C^1 paths.

- **7.2** Line integral of vector fields $\int_{\boldsymbol{c}} \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{s} = \int_a^b \boldsymbol{F}(\boldsymbol{c}(t)) \cdot \boldsymbol{c}'(t) \ \mathrm{d}t$

  Work; Differential form of a line integral; Line integrals over curves with opposite orientation.

- **7.4-5** Surface integral of scalar function $\iint_S f \ \mathrm{d}S = \iint_D f(\boldsymbol{\Phi}(u,v)) \ ||\boldsymbol{T}_u \times \boldsymbol{T}_v|| \ \mathrm{d}u \ \mathrm{d}v$
  Area of a surface; Mass of a surface.

- **7.6** Surface integral of vector field $\iint_S \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{S} = \iint_S \boldsymbol{F} \cdot \boldsymbol{n} \ \mathrm{d}S = \iint_D \boldsymbol{F}(\boldsymbol{\Phi}(u,v)) \cdot (\boldsymbol{T}_u \times \boldsymbol{T}_v) \ \mathrm{d}u \ \mathrm{d}v$

  Heat flux; Flow rate; Surface integrals over surfaces with opposite orientation.

- **8.1** Green for $\boldsymbol{F} = (P, Q)$: $\int_{\partial D} \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{s} = \int_{\partial D} P \ \mathrm{d}x + Q \ \mathrm{d}y = \iint_D (\partial Q/\partial x - \partial P/\partial y) \ \mathrm{d}x \ \mathrm{d}y$

  Closed curves; Boundary of a 2D region; Area of a surface.

- **8.2** Stokes: $\iint_S (\boldsymbol{\nabla} \times \boldsymbol{F}) \cdot \mathrm{d}\boldsymbol{S} = \int_{\partial S} \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{s}$; Boundary of a surface; Closed surface.

- **7.2/8.3** Line integral of gradient field $\boldsymbol{F} = \boldsymbol{\nabla} f$: $\int_{\boldsymbol{c}} \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{s} = \int_{\boldsymbol{c}} (\boldsymbol{\nabla} f) \cdot \mathrm{d}\boldsymbol{s} = f(\boldsymbol{c}(b)) - f(\boldsymbol{c}(a))$

  Conservative field; Scalar potential of a gradient field.

- **8.4** Gauss: $\iiint_W \boldsymbol{\nabla} \cdot \boldsymbol{F} \ \mathrm{d}V = \iint_{\partial W} \boldsymbol{F} \cdot \boldsymbol{n} \ \mathrm{d}S$; Orientation of closed boundary surface ∂W of a 3D solid;

  Unit outward normal $\boldsymbol{n}$.

- **8.4/8.1** Gauss in $\mathbb{R}^2$ $\iint_D \boldsymbol{\nabla} \cdot \boldsymbol{F} \ \mathrm{d}A = \int_{\partial D} \boldsymbol{F} \cdot \boldsymbol{n} \ \mathrm{d}s$; Closed curve ∂D; Outward unit normal $\boldsymbol{n}$.

- **4.4/8.2/8.4** Interpretation of divergence and curl.

What not to know (everything we did **not** discuss in class):

- Integration by parts; Applications involving electric fields; Historical Notes.

- **7.1-4**: The Total Curvature of a Curve (p.355-356); Line Integrals over Geometric Curves (p.368-370); The $\mathrm{d}\boldsymbol{r}$ Notation for Line Integrals (p.371-373); Formula (4) on p.387; Formula (6) on p. 388.

- **8.1-4**: Theorem 3 (p.434); p.447 Ex. 4 to p.450; Theorem 8 (p.459); Divergence in Spherical Coordinates (p.470-471).