Math 3214: Homework 7 (Due Friday 3/28, 5pm)

To obtain (full) credit, show all reasoning and work.

No calculator or other electronic devices for HWs.

1. Let E be the parallelepiped spanned by the vectors $\overrightarrow{OA} = (1,1,0)$, $\overrightarrow{OB} = (1,1,2)$, and $\overrightarrow{OC} = (2,0,2)$. Evaluate $\iiint_E (x-y)\, dV$ using an appropriate change of variables. Include a sketch of E and E^*.

2. Section 4.2: 1. Include the formula for arc length.

3. Section 7.1: 11b. Include the formula for a path integral.

4. Section 7.2: 3d. Include the formula for a line integral.

5. Section 7.2: 6ab. Clearly show all work and reasoning.

6. Section 7.2: 9.

Problems 7-11 require an appropriate sketch of the curve represented by $c(t)$.

7. Let $f(x,y) = x + y$ and $c(t) = (t^4,\ t^4)$ with $-1 \le t \le 1$.

   (a) Compute $\displaystyle\int_c f\, ds$.

   (b) Find the answer (number) you obtained in (a) geometrically.

8. Compute the mass of the wire formed by $(x-1)^2 + (y-1)^2 = 4$. The density is $\delta(x,y) = 2+x$.

9. Let C be the curve of intersection of the surfaces $x = 3y^2$ and $z = 2y^3$. A particle moves along C from $(0,0,0)$ to $(12,2,16)$ and is acted upon by a force field $\boldsymbol{F}(x,y,z) = (2z,\ x,\ y^2)$. Compute the work done by the force field.

10. Let $\boldsymbol{F}(x,y) = (y^2,\ x)$ and let C be the the perimeter of the triangle with vertices $(0,0)$, $(1,0)$, and $(1,2)$.

    (a) Find $\displaystyle\int_C \boldsymbol{F}\cdot\, d\boldsymbol{s}$ when the orientation of C is counter-clockwise.

    (b) Find $\displaystyle\int_C y^2\, dx + x\, dy$ when the orientation of C is clockwise.

11. Let $f(x,y,z) = 6yz$ and let C be the part of the curve of intersection of $x^2 + z^2 = 4$ and $y = x$ that lies in the first octant.

    (a) Find $\displaystyle\int_C f\, ds$ when C has orientation from $(2,2,0)$ to $(0,0,2)$.

    (b) Find $\displaystyle\int_C f\, ds$ when C has orientation from $(0,0,2)$ to $(2,2,0)$.