Math 3214: Homework 6 (Due Wednesday 3/19, 5pm)

To obtain (full) credit, show all reasoning and work.

No calculator or other electronic devices for HWs.

1. Section 6.1: 1a.
2. Section 6.1: 1b.
3. Section 6.1: 3.
4. Section 6.1: 7. Prove that $\boldsymbol{T}$ is one-to-one and also sketch $D = \boldsymbol{T}(D^*)$.
5. Let $D^* = [0,1] \times [0,1]$ and $\boldsymbol{T}(u,v) = (u,\ u^2 + v^2)$.
   (a) Find and sketch $D = \boldsymbol{T}(D^*)$. Find each boundary curve in terms of x and y only.
   (b) Prove that $\boldsymbol{T}$ is one-to-one.
6. Let $D^* = [0,1] \times [0,1]$ and $\boldsymbol{T}(u,v) = (uv,\ 1)$.
   (a) Find and sketch $D = \boldsymbol{T}(D^*)$. Find each boundary curve in terms of x and y only.
   (b) Prove that $\boldsymbol{T}$ is not one-to-one.
7. Section 6.2: 1b.
8. Section 6.2: 19. Use an appropriate change of variables and briefly explain why your change of variables can be used.

For problems 9-11 you don't need to check the conditions of the Change of Variables Theorem

9. Section 6.2: 35b.
10. Let D be the region in the first quadrant bounded by $x = 0$, $y = x$ and $y = 1 - x^2 + x$. Compute

    $\iint_D x\, \mathrm{e}^{(y+x^2-x)^2}\, \mathrm{d}x\, \mathrm{d}y$ using the change of variables $x = v$ and $y = u^2 + v$ with $u \geq 0$.
11. Let D be the region bounded by $x = \sqrt{y}$, $y = 0$, $y = 1 - x^2$, and $y = 2 - x^2$. Compute

    $\iint_D x \cos\left(\dfrac{x^2 - y}{x^2 + y}\right)\, \mathrm{d}x\, \mathrm{d}y$ using the change of variables $u = x^2 - y$ and $v = x^2 + y$.