Math 3214: Homework 5 (Due Wednesday 3/5, 5pm)

To obtain (full) credit, make appropriate sketches of D and E

No calculator or other electronic devices for HWs.

Problems 1-5 need a good sketch of D. Also shade D.

1. Let D be a thin plate in $\mathbb{R}^2$ enclosed by $y = x^2$ and $y = 2x + 3$. Set up a double integral to compute the area A of the lamina in both $\mathrm{d}x\,\mathrm{d}y$ and $\mathrm{d}y\,\mathrm{d}x$ order. Include the formula for A.
2. Section 5.4: 1c.
3. Section 5.4: 5.
4. Section 6.2: 11.
5. Set up a double integral for $\iint_D x\,\mathrm{d}A$ in $\mathrm{d}x\,\mathrm{d}y$ order, $\mathrm{d}y\,\mathrm{d}x$ order, or polar coordinates. **Briefly explain** why the double integral is easier to set up for your choice.
   (a) D is the region in the first quadrant enclosed by $x^2 + y^2 = 4$, $x = 1$, $y = 0$, and $x = 0$.
   (b) D is enclosed by $y = \sqrt{4 - x^2}$, $y = x\sqrt{3}$, and $y = -x$.
   (c) D is enclosed by $y = x$, $y = 2$, and $x + y = 2$.

Problems 6-10 need a good sketch of solid E and 2D region D for the outer double integral.

6. Let E be the solid enclosed by the xz-plane, yz-plane, $z = x + 2y$ and $z = 2$. Set up a triple integral in $\mathrm{d}z\,\mathrm{d}x\,\mathrm{d}y$ order to integrate $f(x, y, z) = x^2$.
7. Set up a triple integral in rectangular coordinates for the volume of the solid enclosed by $x = y^2 + 4z^2$ and $x = 8 - y^2 - 4z^2$. Choose an easy order for the setup and briefly explain why your choice is the easiest order.
8. Set up a triple integral in either cylindrical or spherical coordinates for $\iiint_E x^2 z\,\mathrm{d}V$ and briefly explain why the triple integral is easier to set up and/or evaluate for your choice.
   (a) E is inside $x^2 + y^2 + z^2 = 4$ and between $z = -\sqrt{x^2 + y^2}$ and $z = \sqrt{3x^2 + 3y^2}$.
   (b) E is enclosed by $z = 12 - x^2 - y^2$ and $z = \sqrt{x^2 + y^2}$.
9. Compute the volume V of the solid enclosed by $z = x^2 + y^2$ and $z = 4$. Include the formula for V.
10. Compute $\iiint_E 6z^5\,\mathrm{d}V$ where E is the tetrahedron with vertices $(0, 0, 0)$, $(2, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 2)$.