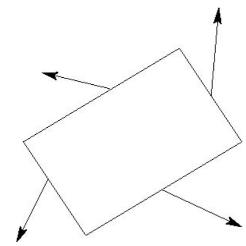
Math 3214: Homework 11 (Due Friday 4/25, 5pm)

To obtain (full) credit, show all reasoning and work.

No calculator or other electronic devices for HWs.

1. Consider the small rectangular region on the right with velocity vectors $\boldsymbol{v}(x, y)$ indicated at the boundary. The vectors shown are to scale and representative for the vector field along each boundary.



   (a) Explain, using an integral theorem, whether $\boldsymbol{\nabla} \cdot \boldsymbol{v}$ is clearly positive, clearly negative, or could be zero.

   (b) Explain, using an integral theorem, whether the scalar curl is clearly positive, clearly negative, or could be zero.

Problems 2-8: Include an appropriate sketch with the orientation of each curve or surface.
Include the orientation in your sketch and briefly explain.
Include the Ch. 7 and Ch. 8 formulas you used in each problem.

2. Compute $\int_{\boldsymbol{c}} \boldsymbol{F} \cdot \, \mathrm{d}\boldsymbol{s}$, where $\boldsymbol{F}(x, y, z) = (2xyz + \sin x,\ x^2 z,\ x^2 y)$ and $\boldsymbol{c}(t) = (\cos^5 t,\ \sin^3 t,\ t^4)$ with $0 \le t \le \pi$.

3. Compute $\iint_S \boldsymbol{F} \cdot \boldsymbol{n} \, \mathrm{d}S$ where $\boldsymbol{F}(x, y, z) = \big(y,\ 1,\ z(x^2 + y^2)^2\big)$, $\boldsymbol{n}$ is the outward pointing unit normal, and S is the surface of the solid cylinder $x^2 + y^2 \le 1$ with $0 \le z \le 1$.

4. Let $\boldsymbol{F}(x, y, z) = (5,\ xz,\ \mathrm{e}^{z^5})$ and $\boldsymbol{c}$ consist of the 4 straight lines joining $(0, 0, 0)$, $(1, 2, 0)$, $(1, 2, 1)$, and $(0, 0, 1)$, in this order. Compute $\int_{\boldsymbol{c}} \boldsymbol{F} \cdot \, \mathrm{d}\boldsymbol{s}$.

5. Compute $\iint_S z^2 \, \mathrm{d}S$ where S is the sphere with radius 3.

6. Let $\boldsymbol{F}(x, y) = (\sin(x^2),\ x + \mathrm{e}^{y^2})$ and $\boldsymbol{c}$ consist of the 3 straight lines joining $(0, 0)$, $(1, 0)$, and $(0, 1)$, in this order. Compute $\int_{\boldsymbol{c}} \boldsymbol{F} \cdot \, \mathrm{d}\boldsymbol{s}$.

7. Compute $\int_{\boldsymbol{c}} \boldsymbol{F} \cdot \boldsymbol{n} \, \mathrm{d}s$ where $\boldsymbol{c}$ corresponds to $y = x^2$ from $(0, 0)$ to $(1, 1)$ and $\boldsymbol{n}$ the unit normal pointing in the positive y-direction. $\boldsymbol{F}(x, y) = (1,\ -2x)$.

8. Compute the heat flux through $x^2 + y^2 + z^2 = 9$. $T(x, y, z) = xy + yz + xz + z^2$ and $k = 1$.