Prerequisites: Extrema for Smooth Functions $f(x)$

Procedure: Local extrema

1. Find all critical points: all points where $\mathrm{d}f/\mathrm{d}x = 0$
2. Use 2nd derivative test for each critical point
   (a) $\mathrm{d}^2 f/\mathrm{d}x^2 > 0$: local minimum (function is increasing since $\mathrm{d}f/\mathrm{d}x = 0$)
   (b) $\mathrm{d}^2 f/\mathrm{d}x^2 < 0$: local maximum (function is decreasing since $\mathrm{d}f/\mathrm{d}x = 0$)
   (c) $\mathrm{d}^2 f/\mathrm{d}x^2 = 0$: inconclusive

Example: Extreme values for $f(x) = x^2$

1. Critical points: $\mathrm{d}f/\mathrm{d}x = 2x = 0$, which gives $x = 0$
2. 2nd derivative test: $\mathrm{d}^2 f/\mathrm{d}x^2 = 2$

   $\mathrm{d}^2 f/\mathrm{d}x^2(x = 0) = 2 > 0$: Local minimum $f(0) = 0$

   No local maximum ($x = 0$ is the only critical point)

Prerequisites: Extrema for Smooth Functions $f(x)$

Procedure: Absolute maximum and minimum on a finite, closed interval

1. Find candidates for absolute maxima and minima
   (a) Critical points, INSIDE the region, where $\mathrm{d}f/\mathrm{d}x = 0$
   (b) Boundary points (endpoints)
2. Find absolute max and min: Largest and smallest f value in candidates

Example: Extreme values for $f(x) = 2x^2 - 2x$ on $0 \leq x \leq 2$

1. Candidates
   (a) Critical points: $\mathrm{d}f/\mathrm{d}x = 0$, gives $4x - 2 = 0$, thus $x = 1/2$; Inside $[0, 2]$
   (b) Boundary points: $x = 0$ and $x = 2$
2. Function values in candidates: $f(1/2) = -1/2$; $f(0) = 0$; $f(2) = 4$
   Absolute maximum: $f(2) = 4$ (largest function value)
   Absolute minimum: $f(1/2) = -1/2$ (smallest function value)