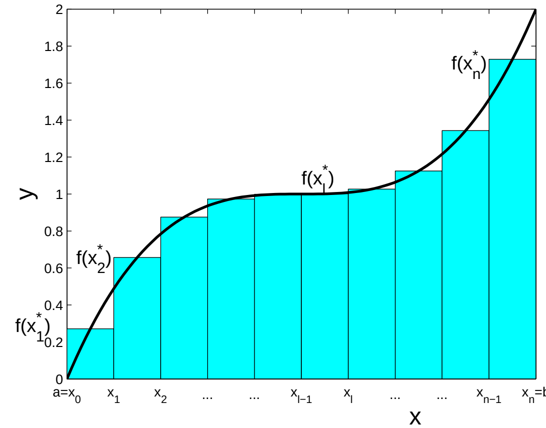
Prerequisites: $\int_a^b f(x)\ dx$

■ Limit definition: $\int_a^b f(x)\ dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$

- $\Delta x = \frac{(b-a)}{n}$: interval length
- x_i^*: Sample point on subinterval $[x_{i-1}, x_i]$
- Interpretation: Area under curve $y = f(x)$



■ Approximation: Midpoint rule $\int_a^b f(x)\ dx \approx \sum_{i=1}^{n} f(x_i^*)\Delta x$

- n: finite number of subintervals
- $x_i^* = \frac{x_{i-1} + x_i}{2}$: midpoint of each subinterval $[x_{i-1}, x_i]$

Prerequisites: Evaluating $\int f(x)\ \mathrm{d}x$

- u-substitution: $\int f(x)\ \mathrm{d}x = \int f(x(u)) \frac{\mathrm{d}x}{\mathrm{d}u}\ \mathrm{d}u$

  Example: $\int x \mathrm{e}^{x^2}\ \mathrm{d}x$

  $u = x^2$ and $\mathrm{d}u = 2x\ \mathrm{d}x$ gives

  $$\int x \mathrm{e}^{x^2}\ \mathrm{d}x = \int \mathrm{e}^u\ \frac{1}{2}\ \mathrm{d}u = \frac{1}{2}\mathrm{e}^u + C = \frac{1}{2}\mathrm{e}^{x^2} + C$$

  Note: for definite integrals, change to u-bounds!

- Recognize too hard integrals: Try to solve differently

  Examples: $\int \mathrm{e}^{x^2}\ \mathrm{d}x$, $\int \sin x^2\ \mathrm{d}x$, etc.