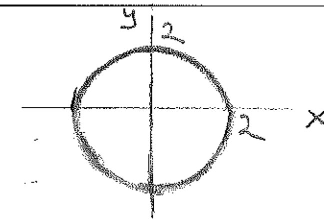
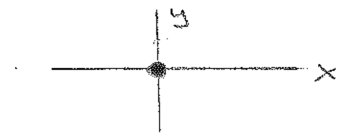
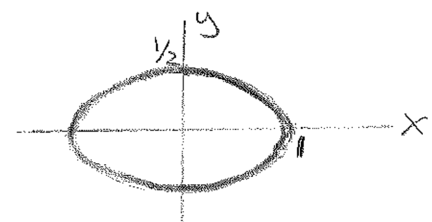
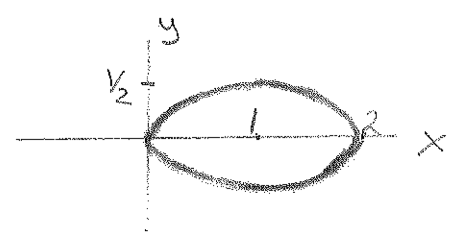
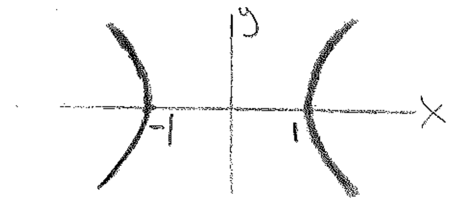
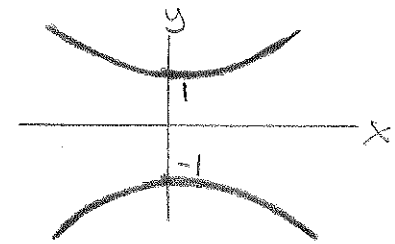
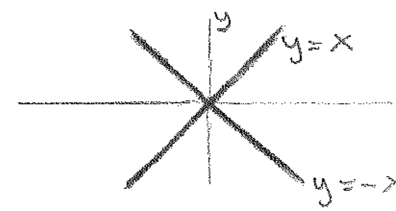
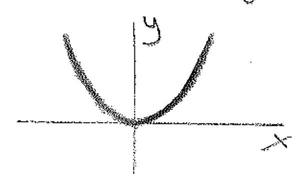
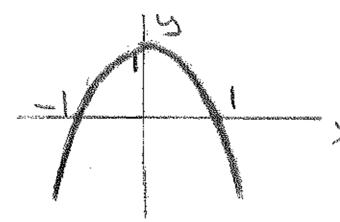
Prerequisites: Curves in 2D

Equation	Shape	Curve
$x^2 + y^2 = 4$	Circle, center $(0, 0)$, radius $\sqrt{4} = 2$	
$x^2 + y^2 = 0$	Single point; $x = 0$ and $y = 0$	
$x^2 + y^2 = -4$	No solution; $x^2 + y^2 \geq 0$	
$x^2 + 4y^2 = 1$	Ellipse, center $(0, 0)$, x-intercepts at $x^2 = 1$, y-intercepts at $4y^2 = 1$	
$(x-1)^2 + 4y^2 = 1$	Shifted ellipse, center $(1, 0)$; x-intercepts at $(x-1)^2 = 1$; y-intercepts at $4y^2 = 1$	

Prerequisites: Curves in 2D

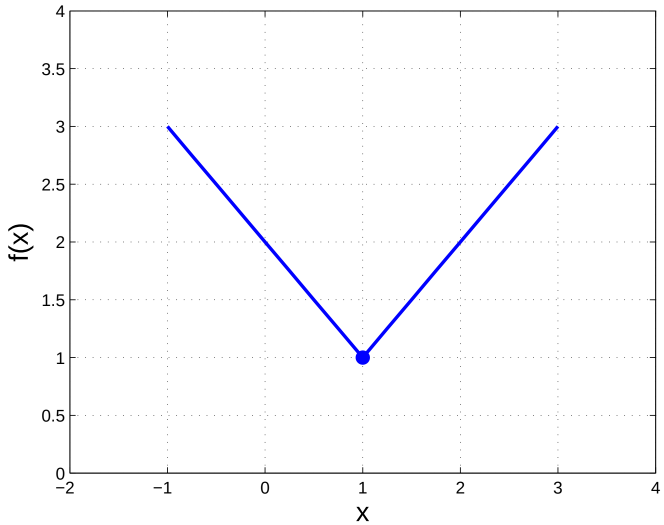
Equation	Shape	Curve
$x^2 - y^2 = 1$	Hyperbola, no solution for $-1 < x < 1$, x-intercepts at $x^2 = 1$	
$x^2 - y^2 = -1$	Hyperbola, no solution for $-1 < y < 1$, y-intercepts at $y^2 = 1$	
$x^2 - y^2 = 0$	Two straight lines, $y^2 = x^2$ or $y = \pm x$	
$y = x^2$	Parabola (up), minimum at $(0, 0)$	
$y = -x^2 + 1$	Parabola (down), maximum at $(0, 1)$	

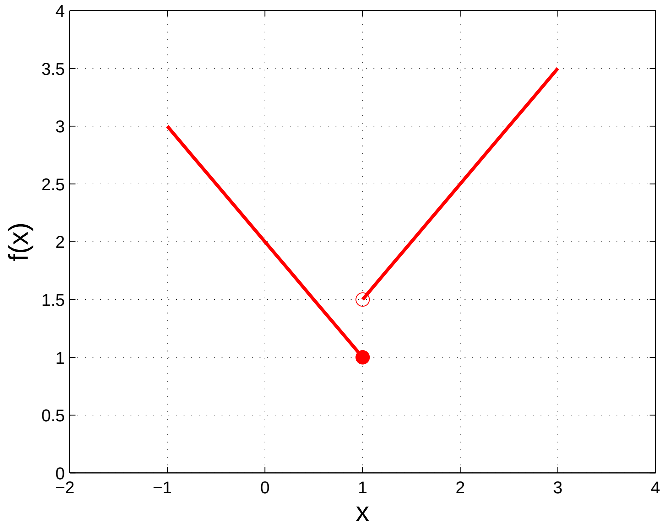
Prerequisites: Domain and Range of $f(x)$

Function	Domain D	Range R
$f(x) = 2$	$f(x)$ defined for all x values thus D is $(-\infty, \infty)$	$f(x)$ can only reach the value 2, thus R is $\{2\}$
$f(x) = 2\sin x$	$f(x)$ defined for all x values thus D is $(-\infty, \infty)$	Since $-1 \leq \sin x \leq 1$, we have R is $[-2, 2]$
$f(x) = \dfrac{1}{x-1}$	$f(x)$ defined for all x, except when $x - 1 = 0$; Thus D is $(-\infty, 1)$ and $(1, \infty)$	On $(-\infty, 1)$: $-\infty < f(x) < 0$; On $(1, \infty)$: $0 < f(x) < \infty$; Thus R is $(-\infty, 0)$ and $(0, \infty)$
$f(x) = \sqrt{x+2}$	$f(x)$ defined for $x + 2 \geq 0$ thus $x \geq -2$; Thus D is $[-2, \infty)$	On D, we have $0 \leq x + 2 < \infty$ which implies $0 \leq \sqrt{x+2} < \infty$; Thus R is $[0, \infty)$

Prerequisites: Limits for Functions $f(x)$

$\lim_{x \to a} f(x)$ exists if you see the same function value when approaching $x = a$ from all directions (from left and right for $f(x)$)





$\lim_{x \to a} f(x)$ does not exist if you see different function values when approaching $x = a$ from different directions (from left and right for $f(x)$)

Prerequisites: Limits for Functions $f(x)$

- **Limits for indeterminate forms "0/0" and "$\pm\infty/\pm\infty$"**

(1) l'Hospital's rule: $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$

Example: $\lim_{x\to 0}\frac{\sin x}{x} = \lim_{x\to 0}\frac{\mathrm{d}\sin x/\mathrm{d}x}{\mathrm{d}x/\mathrm{d}x} = \lim_{x\to 0}\frac{\cos x}{1} = 1$

(2) Multiply by conjugate

Example: $\lim_{x\to 0}\frac{1-\sqrt{x+1}}{x} = \lim_{x\to 0}\frac{1-\sqrt{x+1}}{x}\,\frac{1+\sqrt{x+1}}{1+\sqrt{x+1}} =$

$$\lim_{x\to 0}\frac{1-(x+1)}{x(1+\sqrt{x+1})} = \lim_{x\to 0}\frac{-x}{x(1+\sqrt{x+1})} = \lim_{x\to 0}\frac{-1}{(1+\sqrt{x+1})} = -\frac{1}{2}$$

Prerequisites: Derivatives of $f(x)$

Function	Rule	Derivative
$f(x) = x\sin(x)$	Product rule: $(uv)' = u'v + uv'$	$f'(x) = \sin(x) + x\cos(x)$
$f(x) = \dfrac{\sin(x)}{x}$	Quotient rule: $\left(\dfrac{u}{v}\right)' = \dfrac{u'v - v'u}{v^2}$	$\left(\dfrac{\sin(x)}{x}\right)' = \dfrac{\cos(x)x - 1\sin(x)}{x^2}$
$f(x) = \sin(x^2)$	Chain rule: $f(y)$ with $y = g(x)$, $\dfrac{\mathrm{d}f}{\mathrm{d}x} = \dfrac{\mathrm{d}f}{\mathrm{d}y} \cdot \dfrac{\mathrm{d}y}{\mathrm{d}x}$	$f = \sin(y)$ with $y = x^2$: $\dfrac{\mathrm{d}f}{\mathrm{d}x} = \cos(y)(2x) = \cos(x^2)\ (2x)$