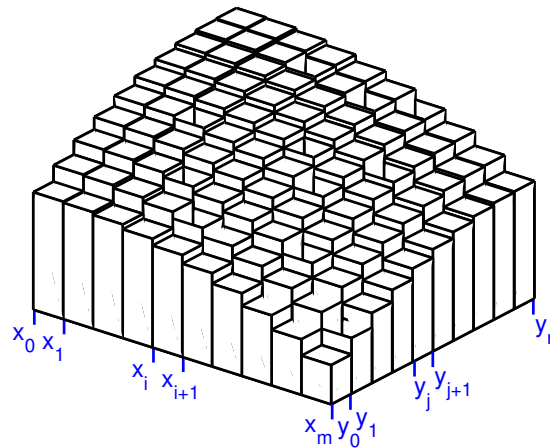
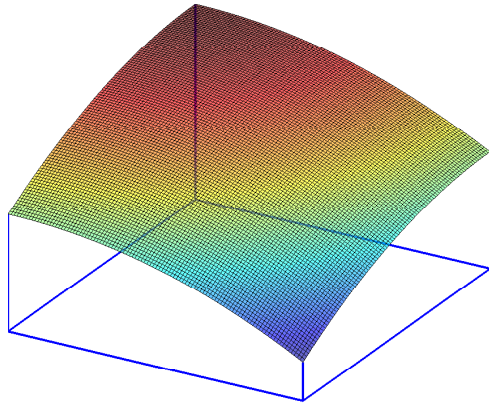


$$15.1: \iint_R f(x, y) \, dA$$



■ Interpretation of $\iint_R f(x, y) \, dA$: Volume under surface $z = f(x, y)$

■ Limit definition:
$$\iint_R f(x, y) \, dA = \lim_{m \rightarrow \infty} \lim_{n \rightarrow \infty} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} f(x_i^*, y_j^*) \Delta A_{ij}$$

- $\Delta A_{ij} = \Delta x_i \Delta y_j = (x_{i+1} - x_i)(y_{j+1} - y_j)$: Area of a subregion
- (x_i^*, y_j^*) : Sample point on subregion $[x_i, x_{i+1}] \times [y_j, y_{j+1}]$
- $f(x_i^*, y_j^*) \Delta A_{ij}$: Volume of one rectangular block

■ Idea: Volume of all rectangular blocks approaches volume under $z = f(x, y)$