14.8: Lagrange Multipliers for Smooth Functions f

- **Extrema** (max/min) of f **subject to a constraint** (side condition) $g = 0$
  - **In** $\mathbb{R}^2$: $g(x, y) = 0$ represents a (level) curve
  - **In** $\mathbb{R}^3$: $g(x, y, z) = 0$ represents a (level) surface

- **Closed and bounded set**: absolute max and min exist

  Typical examples of constraints that form a closed and bounded set
  - **In** $\mathbb{R}^2$: ellipse (the curve)
  - **In** $\mathbb{R}^3$: ellipsoid (the surface)

- **Not closed or not bounded set**: absolute max and/or min may not exist

  Typical examples of constraints that form an unbounded set
  - **In** $\mathbb{R}^2$: parabola, line
  - **In** $\mathbb{R}^3$: paraboloid, plane

14.8: Lagrange Multipliers

Procedure: **Lagrange multipliers for 1 constraint**

If $\underline{\nabla} g \neq \underline{0}$ at all points on the constraint $g = 0$

1. **Find candidates** by solving system of equations

$$\begin{cases} g = 0 & \text{Constraint} \\ \underline{\nabla} f = \lambda \underline{\nabla} g & \text{Lagrange multiplier } \lambda \end{cases}$$

2. **Determine if absolute minimum and/or maximum exist**

3. **Find maximum and/or minimum**: largest and/or smallest f value

Remarks

- $\underline{\nabla} g = \underline{0}$ on $g = 0$: add points where $\underline{\nabla} g = \underline{0}$ to the list of candidates
- Lagrange multiplier λ needs to have a solution: always compute λ!