14.7: Extrema for Smooth Functions $f(x, y)$

Procedure: Local extrema and saddle points

1. Find all Critical Points: $f_x = 0$ and $f_y = 0$

2. Use Second Derivative Test at each CP (a, b)

   $$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{vmatrix} = f_{xx} f_{yy} - (f_{xy})^2$$

   D is the determinant of the matrix with 2nd partial derivatives

   (a) $f(a, b)$ is a local minimum if $D(a, b) > 0$ and $f_{xx} > 0$

   (b) $f(a, b)$ is a local maximum if $D(a, b) > 0$ and $f_{xx} < 0$

   (c) (a, b) is a saddle point if $D(a, b) < 0$

   (d) Second Derivative Test is inconclusive if $D(a, b) = 0$

14.7: Extrema for Smooth Functions $f(x, y)$

Absolute extrema of $f(x, y)$ on a closed and bounded region D

- Closed: D includes all its boundary points
- Bounded: "D does not extend to infinity"

- Absolute minimum: all points on D have a larger or equal f value
- Absolute maximum: all points on D have a smaller or equal f value

Remarks

- The Extreme Value Theorem for $f(x, y)$ (Th. 8), guarantees an absolute max and min exists for closed and bounded regions D and continuous f
- No absolute max and/or min needs to exist if D is not closed or not bounded

  Example $f(x, y) = x^2 + y^2$ on $\mathbb{R}^2$ has an absolute min but no absolute max: $f \to \infty$ as x or $y \to \infty$
- Absolute max/min are also called global max/min

14.7: Extrema for Smooth Functions $f(x, y)$

Procedure: **Absolute extrema on closed and bounded regions**

1. **Find candidates for absolute maxima and minima**

   (a) Critical points on region: $f_x = 0$ and $f_y = 0$

   (b) Critical points on boundary curves $y = Y(x)$ or $x = X(y)$

   (i) Reduce $f(x, y)$ **to function of 1 variable: substitute boundary curve**

   (ii) Find extrema of function of 1 variable: $f(x, Y(x))$ or $f(X(y), y)$

   (c) Endpoints of boundary curves

2. **Find absolute max and min**: **Largest and smallest** f **value in candidates**