14.3: Partial Derivatives

- **Notation for regular derivatives of $f = f(x)$**

  1. $\dfrac{\mathrm{d}f}{\mathrm{d}x}$ Straight d's

  2. f'

- **Notation for partial derivatives of $f = f(x, y, z)$**

  1. $\dfrac{\partial f}{\partial x}$, $\dfrac{\partial f}{\partial y}$, $\dfrac{\partial f}{\partial z}$ Round ∂'s

  2. f_x, f_y, f_z

- **Notation for partial derivatives of $f = f(x_1, \ldots, x_n)$**

  $$\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \cdots, \frac{\partial f}{\partial x_n}$$

14.3: Partial Derivatives

■ **Notation for second partial derivatives** of $f = f(x, y)$

$$f_{xx} = \frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) \qquad f_{xy} = (f_x)_y = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x}$$

$$f_{yy} = \frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) \qquad f_{yx} = (f_y)_x = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x \partial y}$$

■ **Notation for second partial derivatives** of $f = f(x_1, \ldots, x_n)$

$$\frac{\partial^2 f}{\partial x_1^2}, \quad \frac{\partial^2 f}{\partial x_1 \partial x_2}, \cdots, \quad \frac{\partial^2 f}{\partial x_1 \partial x_n} \text{ etc.}$$

■ **Notation for third partial derivatives** of $f = f(x, y, z)$

$$f_{xyz} = \frac{\partial^3 f}{\partial z \partial y \partial x} = \frac{\partial}{\partial z}\left[\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)\right] \text{ etc.}$$

14.3: Partial Derivatives

■ Computing second partial derivatives

- **Example**: Compute second partial derivatives of $f(x,y) = x^2 + xy$

$$f_x = 2x + y \qquad (y \text{ constant})$$

$$f_y = 0 + x \qquad (x \text{ constant})$$

$$f_{xx} = \frac{\partial}{\partial x}(f_x) = \frac{\partial}{\partial x}\Big(2x + y\Big) = 2 \qquad (y \text{ constant})$$

$$f_{xy} = \frac{\partial}{\partial y}(f_x) = \frac{\partial}{\partial y}\Big(2x + y\Big) = 1 \qquad (x \text{ constant})$$

$$f_{yx} = \frac{\partial}{\partial x}(f_y) = \frac{\partial}{\partial x}\Big(x\Big) = 1 \qquad (y \text{ constant})$$

$$f_{yy} = \frac{\partial}{\partial y}(f_y) = \frac{\partial}{\partial y}\Big(x\Big) = 0 \qquad (x \text{ constant})$$