# 13.3: Arc Length

- **Arc length definition**: $L = \int_a^b |\underline{r}'(t)| \, dt$
  - **Meaning**: Length of a curve between points corresponding to $\underline{r}(a)$ and $\underline{r}(b)$
  - $L$ is a non-negative scalar
  - $|\underline{r}'(t)|$ is the speed along the curve (See 13.4)

    "Speed $\times$ time = length", the distance traveled along a curve
- **Arc length function**: $s(t) = \int_a^t |\underline{r}'(u)| \, du$
  - **Meaning**: Length along a curve as a function of $t$
- **Reparametrization w.r.t. arc length**: $\underline{r}(t(s))$
  - **Meaning**: $\underline{r}(t(s))$ is the position after traveling distance $s$ along the curve

# 13.3: Curvature

■ Curvature definition: $\kappa(t) = \left|\dfrac{d\boldsymbol{T}}{ds}\right|$

- Meaning: How fast the curve changes direction
- Large $\kappa$: Rapid change of direction

  Small $\kappa$: Slow change of direction

  Zero curvature (no change in direction): straight line
- Disadvantage: Hard to compute $\kappa$

■ Eq. (10): $\kappa(t) = \dfrac{|\underline{\boldsymbol{r}}'(t) \times \underline{\boldsymbol{r}}''(t)|}{|\underline{\boldsymbol{r}}'(t)|^3}$ Often easiest to compute curvature!

■ Eq. (9): $\kappa(t) = \dfrac{|\underline{\boldsymbol{T}}'(t)|}{|\underline{\boldsymbol{r}}'(t)|}$ Only easy if $|\underline{\boldsymbol{r}}'(t)|$ is constant

■ Curvature for 2D curves $y = f(x)$: $\kappa(x) = \dfrac{|f''(x)|}{\left[1 + (f'(x))^2\right]^{3/2}}$

- Follows from Eq. (10) with $\underline{\boldsymbol{r}}(x) = \langle x, f(x), 0\rangle$, i.e. $z = 0$ (2D) and $t \to x$