13.1: Vector Functions

- **Vector-valued function**: 1 scalar input (typically t) and a vector as output

  Example: Position vector $\underline{r}(t) = \langle x(t),\ y(t),\ z(t)\rangle$

  Assigns to every t in the domain a vector $\underline{r}(t)$ in $\mathbb{R}^3$

- **Domain**: All values of t for which a vector function $\underline{r}(t)$ is defined, i.e. the set of t-values for which all component functions are defined

  Example: $\underline{r}(t) = \langle \sqrt{1-t},\ \ln t\rangle$

  $\sqrt{1-t}$ defined when $t \le 1$ $\qquad$ $\ln t$ defined when $t > 0$

  Combining gives the domain of $\underline{r}(t)$: $0 < t \le 1$

- **Limit**: $\lim_{t\to a} \underline{r}(t) = \left\langle \lim_{t\to a} x(t),\ \lim_{t\to a} y(t),\ \lim_{t\to a} z(t) \right\rangle$

  Example: $\lim_{t\to 0} \left\langle \frac{t}{1-t}, \frac{\sin t}{t} \right\rangle = \left\langle \lim_{t\to 0} \frac{t}{1-t},\ \lim_{t\to 0} \frac{\sin t}{t} \right\rangle = \left\langle \frac{0}{1},\ \lim_{t\to 0} \frac{\cos t}{1} \right\rangle = \langle 0, 1\rangle$

- **Continuous**: $\underline{r}(t)$ is continuous if all component functions are continuous