12.6: Quadric Surfaces

Sketching procedure

1. Write in standard form: complete the square

2. Determine cross-sections with elliptical traces: Find traces with

- **Single points**
- **No solution**
- **Ellipses: include the size of one ellipse**
- **If no elliptical traces exist: find parabolic traces**

3. Construct surface by connecting traces: straight or curved

- **Use another cross-section to determine straight or curved**
- **Name the surface and label coordinate axes for orientation**

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Example 2 $\quad x^2 - y^2 + 4z^2 = 0$

1. **Standard form**: already in standard form

2. **Elliptical traces** in xz: $\quad x^2 + 4z^2 = y^2$

- **Single points**: $y = 0 \quad \rightarrow \quad (0, 0, 0)$

- **No solution**: never

- **Ellipses**: $\quad y^2 > 0 \quad \rightarrow \quad y > 0$ or $y < 0$

  Size of one ellipse: at $y = \pm 1$, x-radius 1 and z-radius 1/2

  Ellipse size increases when $|y|$ increases

3. **Construct surface**

- **Other cross-section**: $z = 0$ gives $x^2 = y^2$ or 2 lines $y = \pm x$, thus straight

- **Shape**: Cone (double cone); Oriented along y-axis

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Example 3 $\quad x^2 - 4y^2 - z^2 = 1$

1. Standard form: already in standard form

2. Elliptical traces in yz: $\quad 4y^2 + z^2 = x^2 - 1$

- **Single points:** $x^2 - 1 = 0$ or $x = \pm 1 \quad \rightarrow \quad (\pm 1, 0, 0)$
- **No solution:** $\quad x^2 - 1 < 0 \quad \rightarrow \quad -1 < x < 1$
- **Ellipses:** $\quad x^2 - 1 > 0 \quad \rightarrow \quad x > 1$ or $x < -1$

  Size of one ellipse: at $x = \pm\sqrt{2}$, y-radius $1/2$ and z-radius 1

  Ellipse size increases when $|x|$ increases

3. Construct surface

- **Other cross-section: $y = 0$ gives hyperbola $x^2 - z^2 = 1$, thus curved**
- **Shape: Hyperboloid of 2 sheets (2 lemon skins); Oriented along x-axis**

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Example 4 $\quad x^2 + y^2/4 + z^2 = 1$

1. **Standard form**: already in standard form

2. **Elliptical traces** in xz: $\quad x^2 + z^2 = 1 - y^2/4$

- **Single points**: $1 - y^2/4 = 0$ or $y = \pm 2 \quad \rightarrow \quad (0, \pm 2, 0)$

- **No solution**: $\quad 1 - y^2/4 < 0 \quad \rightarrow \quad y > 2$ or $y < -2$

- **Ellipses**: $\quad 1 - y^2/4 > 0 \quad \rightarrow \quad -2 < y < 2$

  Largest circle at $y = 0$, radius 1

  Ellipse size decreases when $|y|$ increases

3. **Construct surface**

- **Other cross-section**: $z = 0$ gives ellipse $x^2 + y^2/4 = 1$; thus **curved**

- **Shape**: Ellipsoid (football); **Elongated** in y-direction