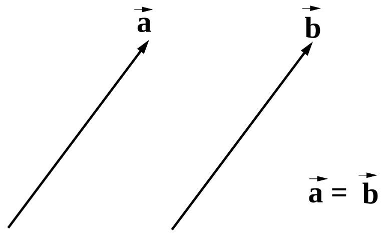
12.2: Vectors

■ **Applications of vectors**: force, velocity, position, acceleration, displacement

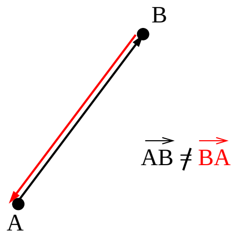
■ **Definition of a vector**: line segment having a direction and magnitude

● Vectors with same direction and length are equal

Ex. People at different locations can have same velocity



● Vectors with opposite direction are not equal



Note: if the starting point of a vector is not specified, it starts at the origin
Ex. Vector to point A is $\overrightarrow{OA}$

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■ Notation for vectors

1. $\boldsymbol{v}$ (bold symbol in printed text)
2. $\vec{v}$ or $\underline{v}$. Never just v which is the scalar (number) v

   Note: $\vec{0}$ is the zero vector (all components zero)

■ Notation for vectors in component form

1. $\vec{v} = \langle v_1, v_2, v_3 \rangle$ in $\mathbb{R}^3$ $\qquad$ $\vec{v} = \langle v_1, v_2 \rangle$ in $\mathbb{R}^2$
2. $\vec{v} = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$ in $\mathbb{R}^3$ $\qquad$ $\vec{v} = v_1\vec{i} + v_2\vec{j}$ in $\mathbb{R}^2$

■ Standard basis vectors: Note the vector notation

1. In $\mathbb{R}^2$: $\vec{i} = \langle 1, 0 \rangle$ and $\vec{j} = \langle 0, 1 \rangle$
2. In $\mathbb{R}^3$: $\vec{i} = \langle 1, 0, 0 \rangle$, $\vec{j} = \langle 0, 1, 0 \rangle$, and $\vec{k} = \langle 0, 0, 1 \rangle$

Alternative notation: $\hat{i}$, $\hat{j}$, and $\hat{k}$

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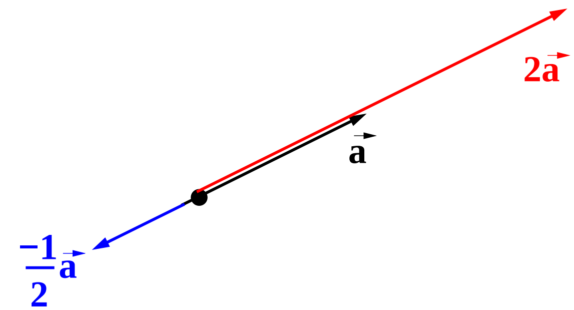
■ **Scalar multiplication**: "Multiply each component"

$$\alpha\vec{a} = \alpha\langle a_1,\ a_2,\ \ldots,\ a_n\rangle = \langle \alpha a_1,\ \alpha a_2,\ \ldots,\ \alpha a_n\rangle$$

Example: $2\langle 1, 2, 3\rangle = \langle 2, 4, 6\rangle$

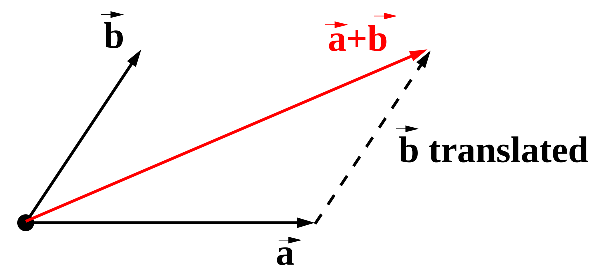


■ **Vector addition**: "Add each component"

$$\vec{a} + \vec{b} = \langle a_1, a_2, \ldots, a_n\rangle + \langle b_1, b_2, \ldots, b_n\rangle$$

$$= \langle a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n\rangle$$

Example: $\langle 1, 2, 3\rangle + \langle 4, 5, 6\rangle = \langle 5, 7, 9\rangle$



■ **Vector subtraction**: "Subtract each component"

$$\vec{a} - \vec{b} = \langle a_1, a_2, \ldots, a_n\rangle - \langle b_1, b_2, \ldots, b_n\rangle$$

$$= \langle a_1 - b_1, a_2 - b_2, \ldots, a_n - b_n\rangle$$

Example: $\langle 1, 2, 3\rangle - \langle 4, 5, 6\rangle = \langle -3, -3, -3\rangle$