PCMI 2018 - Oscillations in Harmonic Analysis
Problem Set #9 on 7/16/2018

P1) Consider

$$I(\lambda) = \int e^{-\pi i \lambda e^{x \cdot (1,1,\ldots,1)}} a(x)dx$$

where $a \in C_c^\infty$ supported in a small enough neighborhood of 0. What can you say about the decay of $I(\lambda)$ in terms of λ where you should only consider λ large?

P2) Consider the proof of the Straightening Lemma we did today.

    (a) Can you remove the condition that $f(p) = 0$?
        [Hint: Apply the theorem to $g(x) = f(x) - f(p)$.]

    (b) Instead of getting a neighborhood U around 0 we got a neighborhood U around a point of the form $(\tilde{p}_1, \ldots, \tilde{p}_{n-1}, 0)$. Can you upgrade the proof so that you move to 0?

P3) In the proof of the Morse Lemma, why can we assume without loss of generality that $p = 0$?