Affine Weyl Groups and Affine Grassmannian Intervals

Michael Lugo

Virginia Tech
Advisor Mark Shimozono

Visitor's Day
17 March 2017

Symmetric Group

- Consider S_3

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$$(1,2,3)$$

$$(2,1,3) \qquad (1,3,2)$$

$$(2,3,1) \qquad (3,1,2)$$

$$(3,2,1)$$

Symmetric Group

- Consider $S_3 = \langle s_1, s_2 \rangle$

$(1, 2, 3)$

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Symmetric Group

- Consider $S_3 = \langle s_1, s_2 \rangle$

1

s_1 s_2

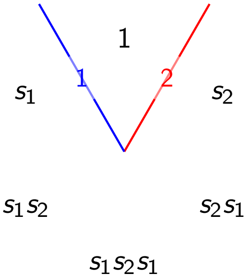
$s_1 s_2$ $s_2 s_1$

$s_1 s_2 s_1$

$s_2 s_1 s_2$

Symmetric Group

- Consider $S_3 = \langle s_1, s_2 \rangle$
- Geometric Interpretation



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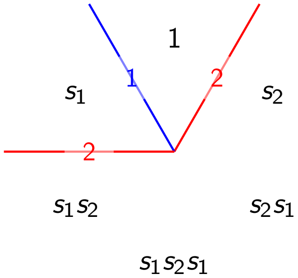

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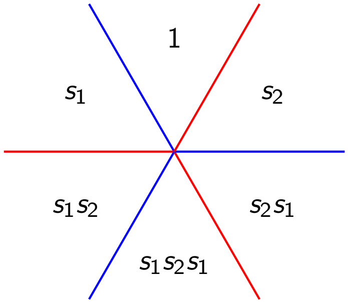

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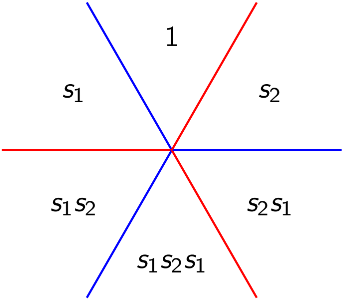
- Walks and $\ell(w)$



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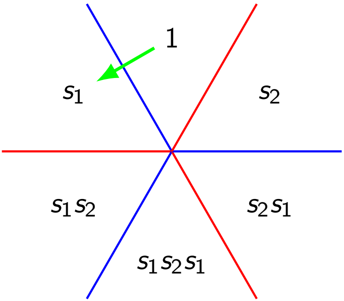

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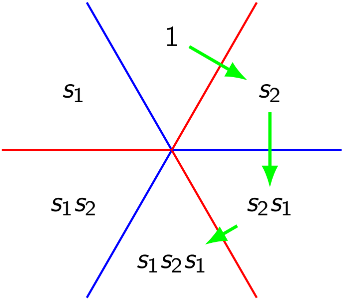

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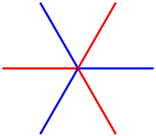

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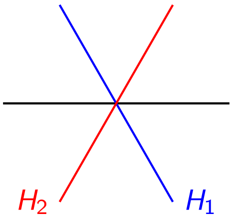

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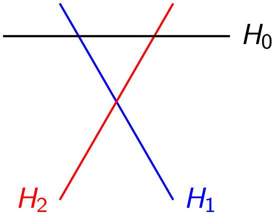

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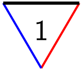

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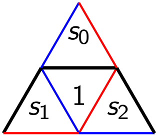

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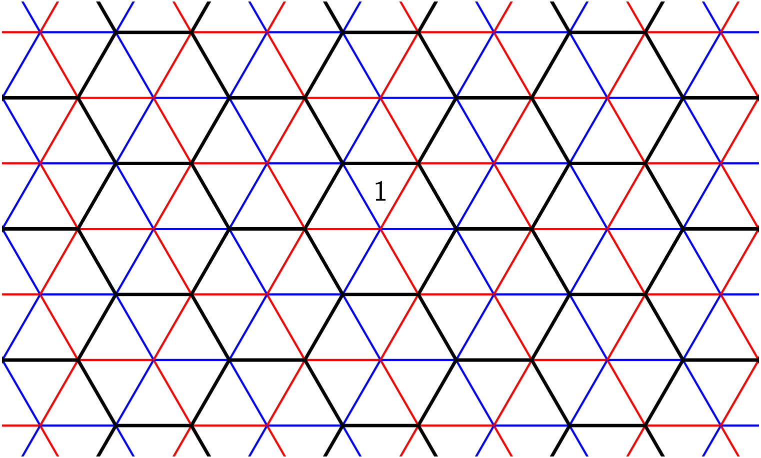
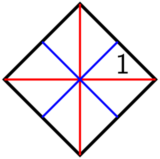


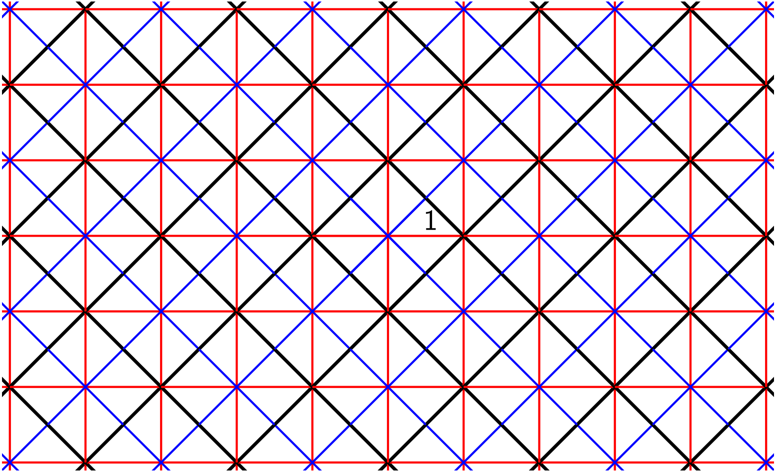
Different Affine Weyl Groups

B_2



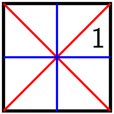
Different Affine Weyl Groups

$\widetilde{B}_2$



Different Affine Weyl Groups

C_2



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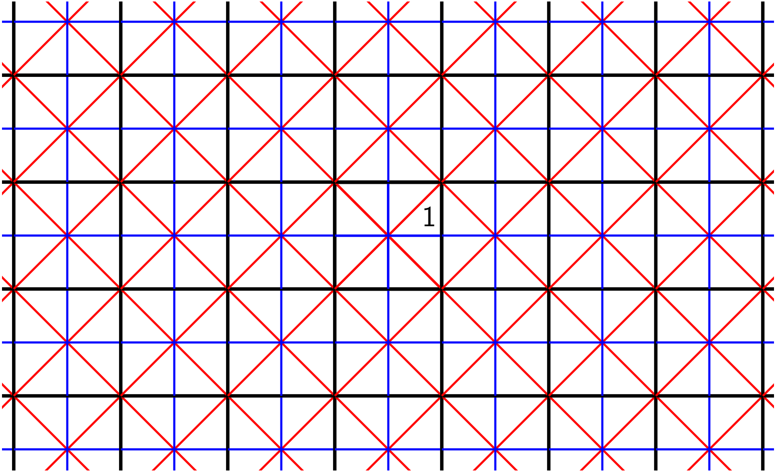
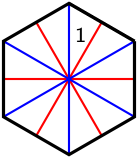


Different Affine Weyl Groups

G_2



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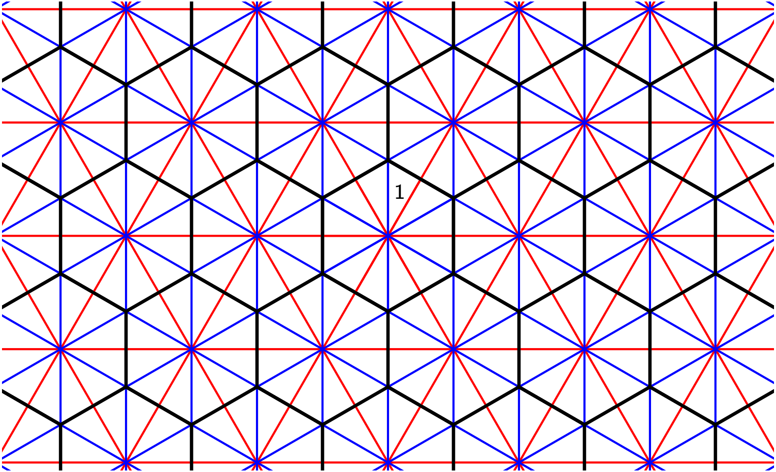
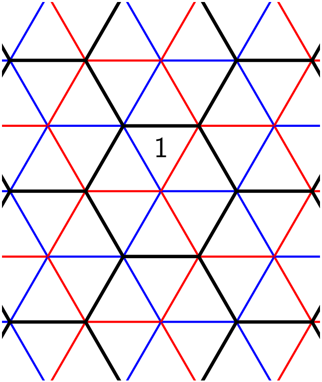


Covers and Intervals

- $u \lessdot v$ if
  - $\ell(v) = \ell(u) + 1$
  - can reflect u to v



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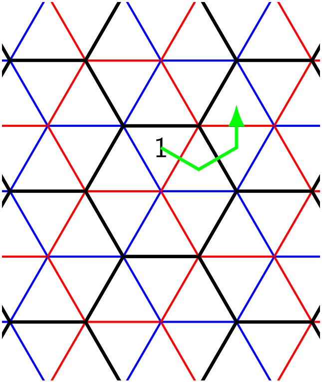

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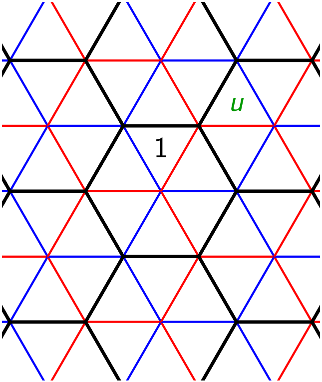

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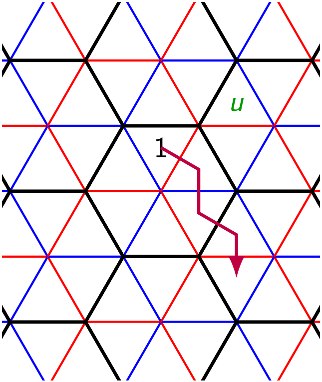

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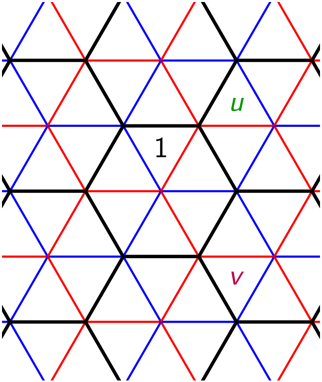

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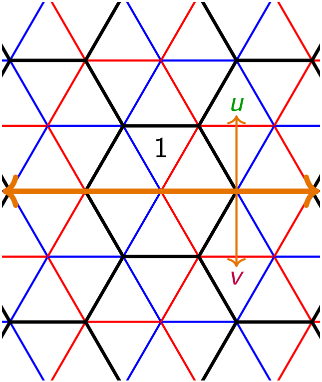

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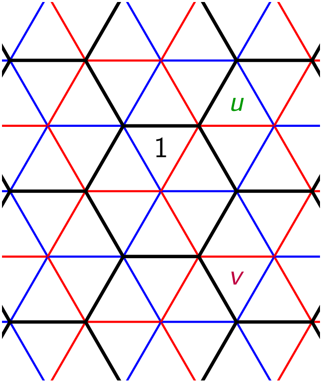

Covers and Intervals

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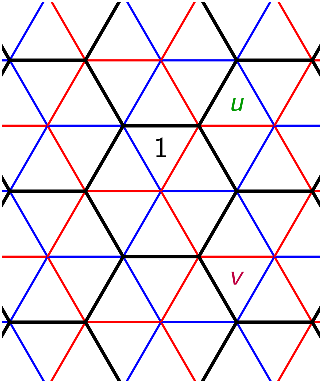
- Extend to partial order
- $[v, w] = \{u \mid v \le u \le w\}$



Affine Grassmannian Elements

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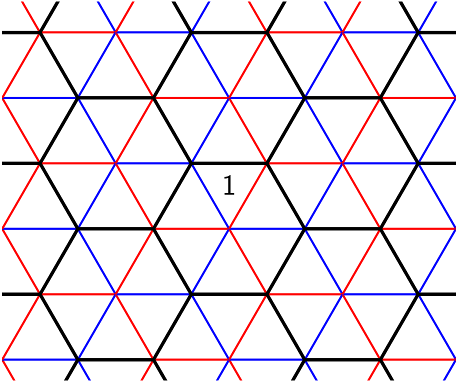

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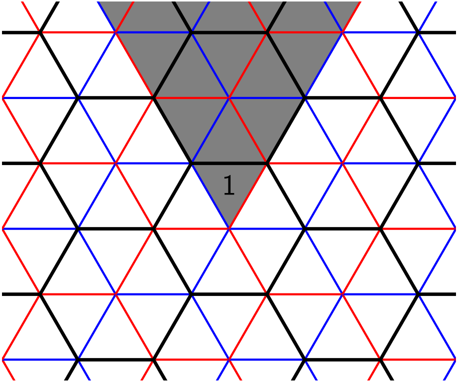

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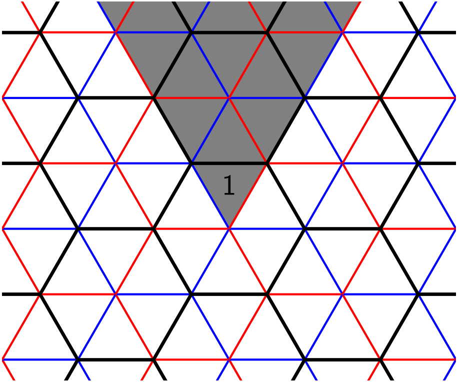

- C is not a group

An Interesting Question

Research Question

If we have a $w \in C$, can we characterize the $v \in C$ such that $[v, w] \subseteq C$?

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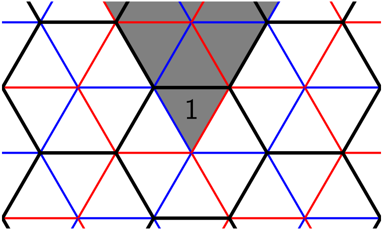

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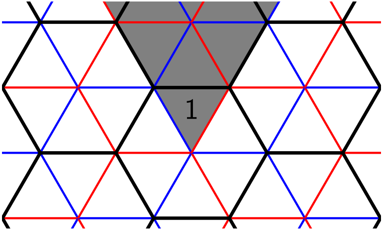

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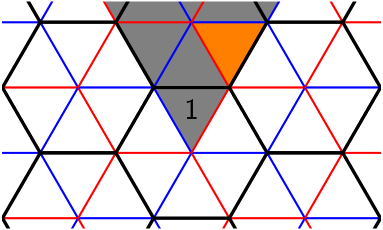

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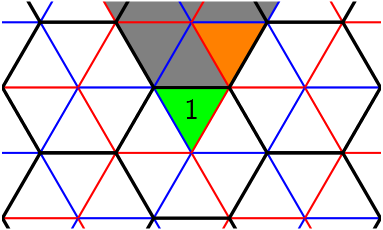

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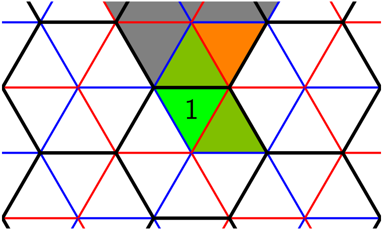

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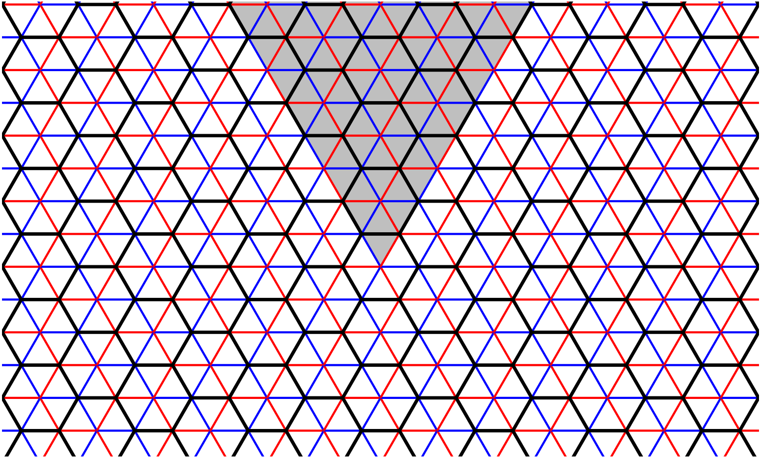

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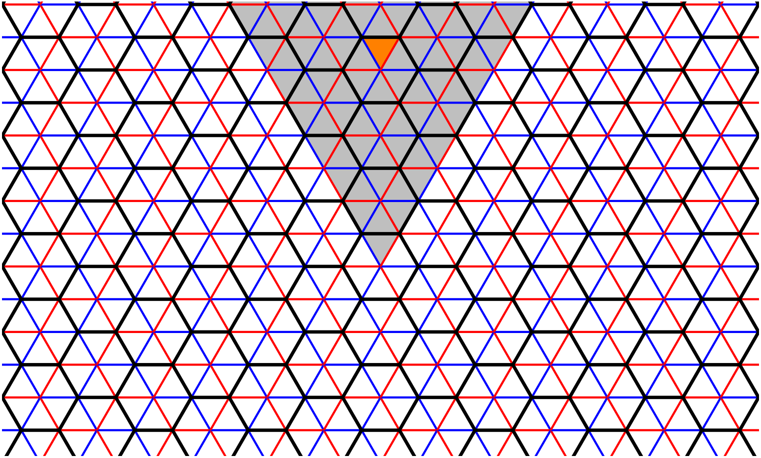

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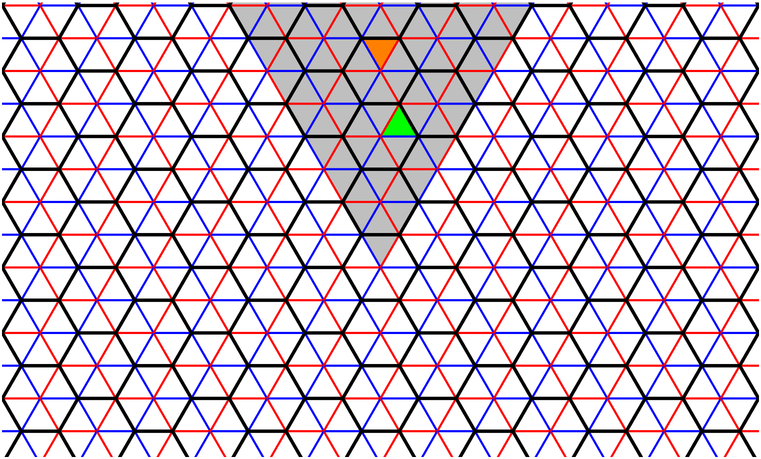

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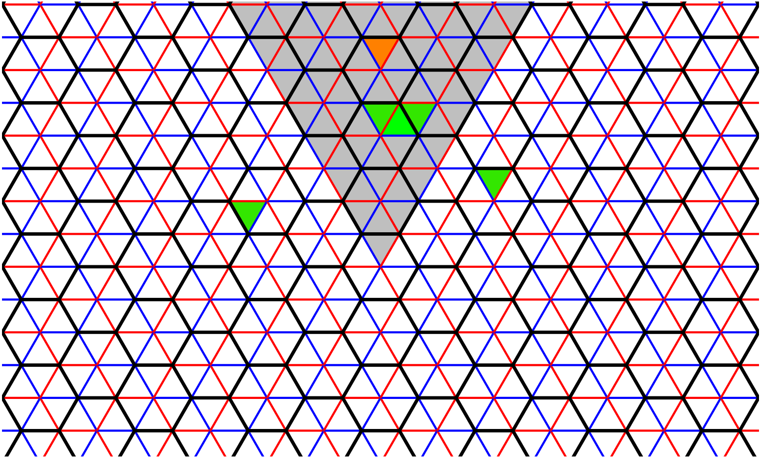

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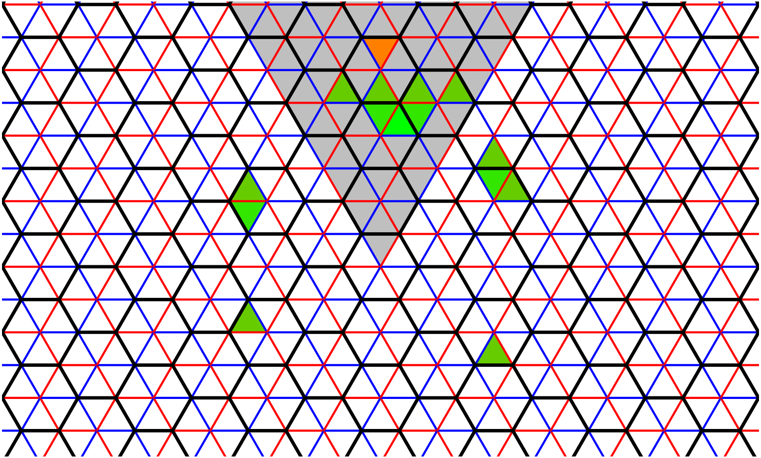

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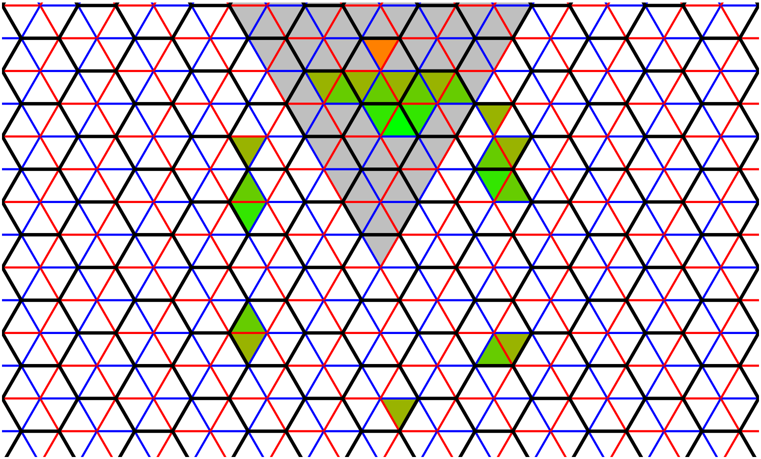

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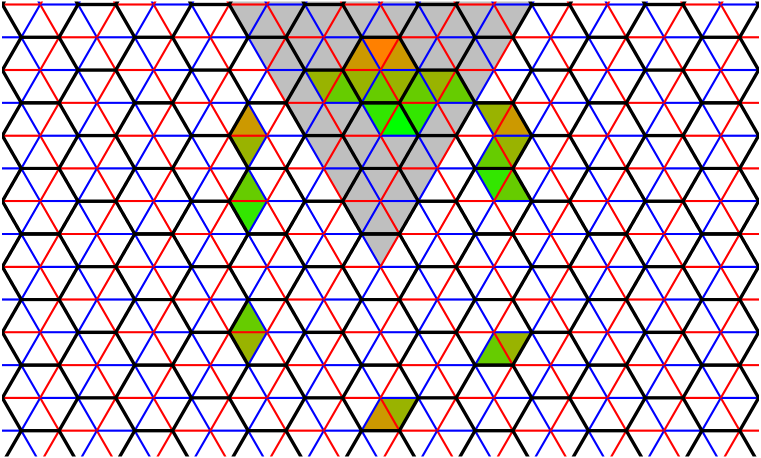

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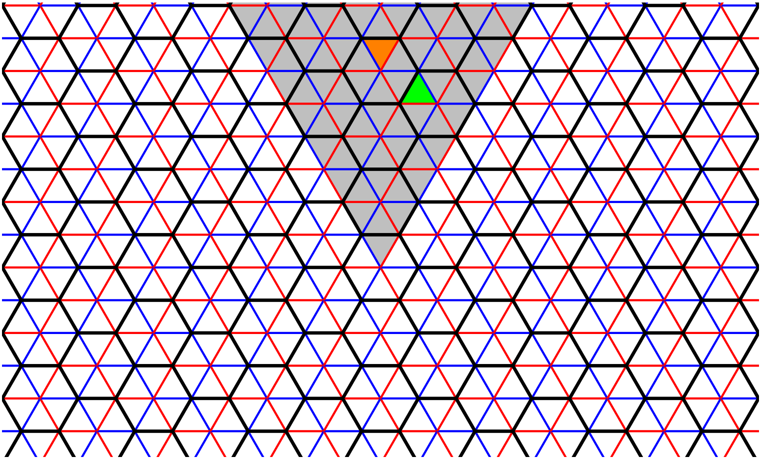

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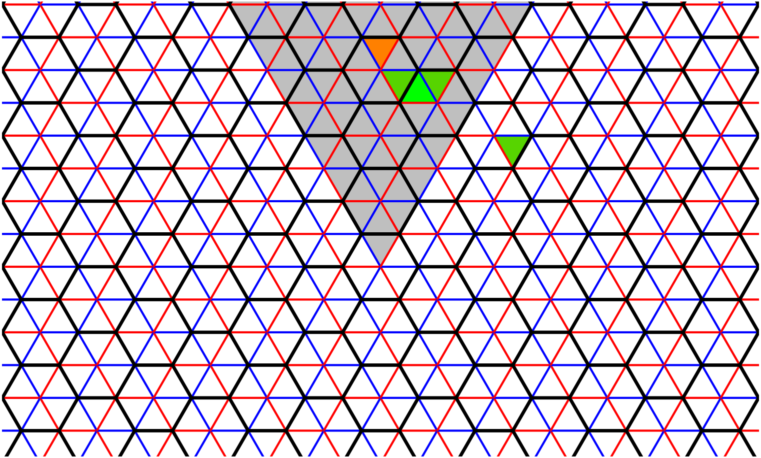

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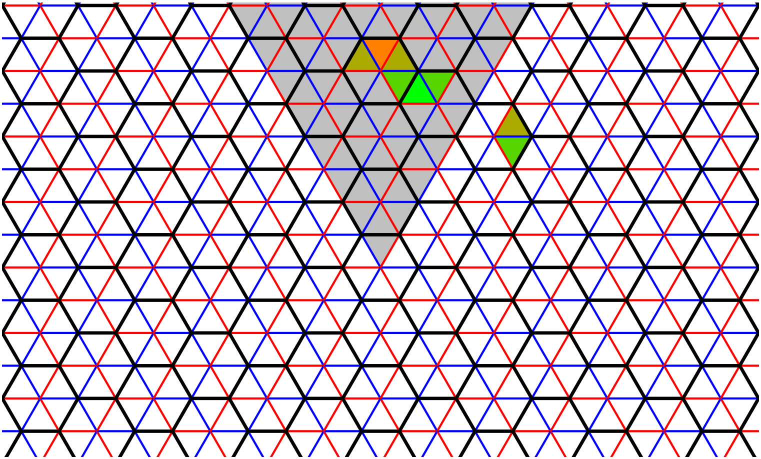

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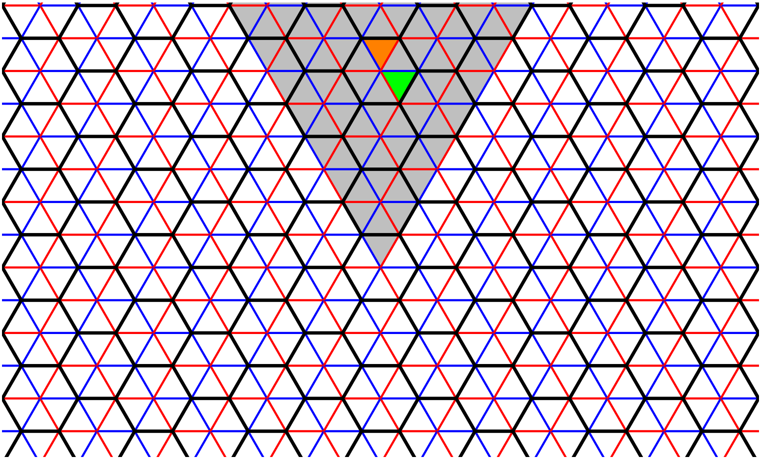

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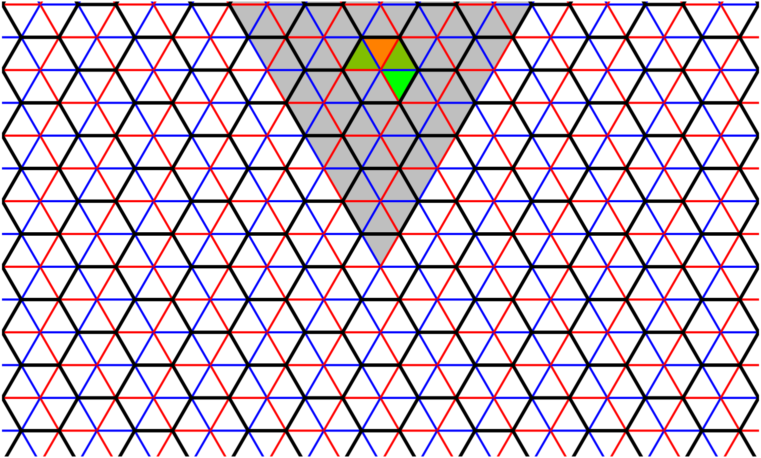
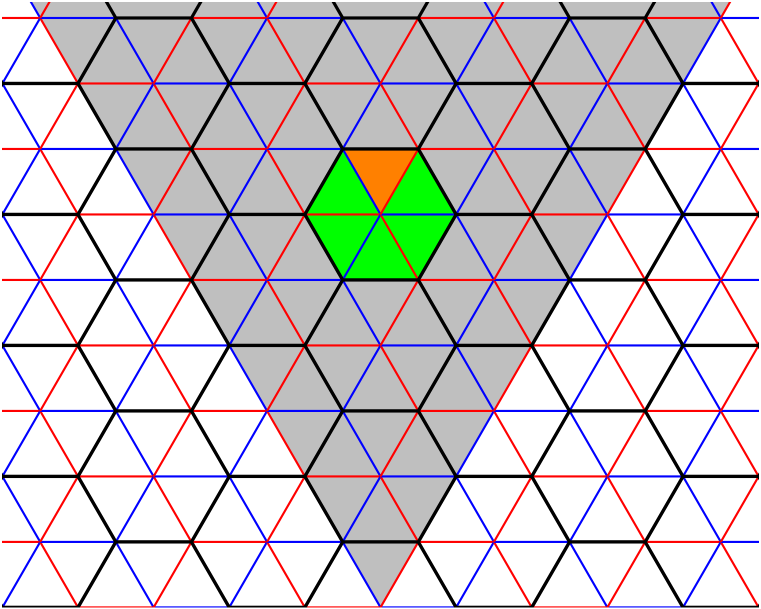


Solutions



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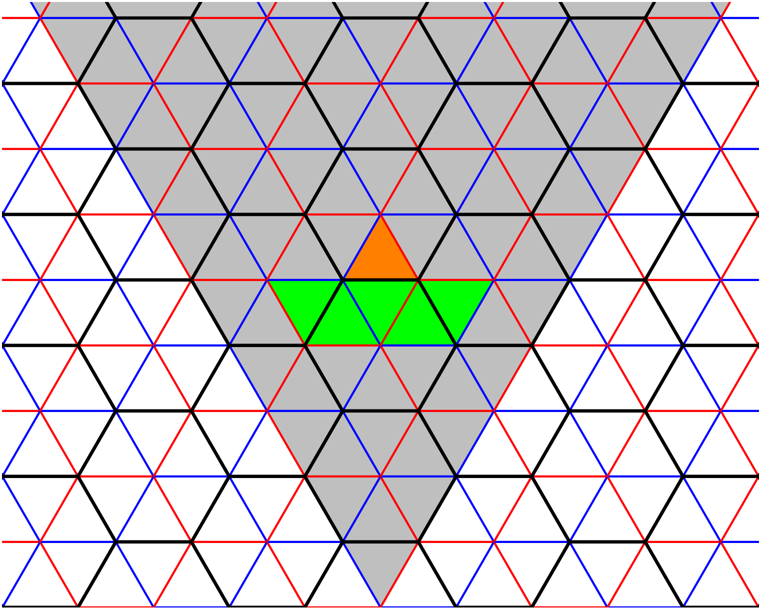

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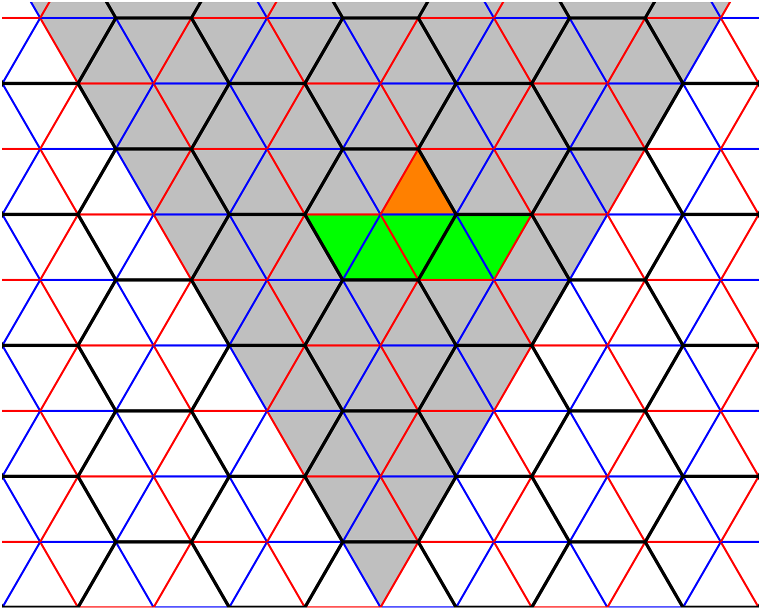

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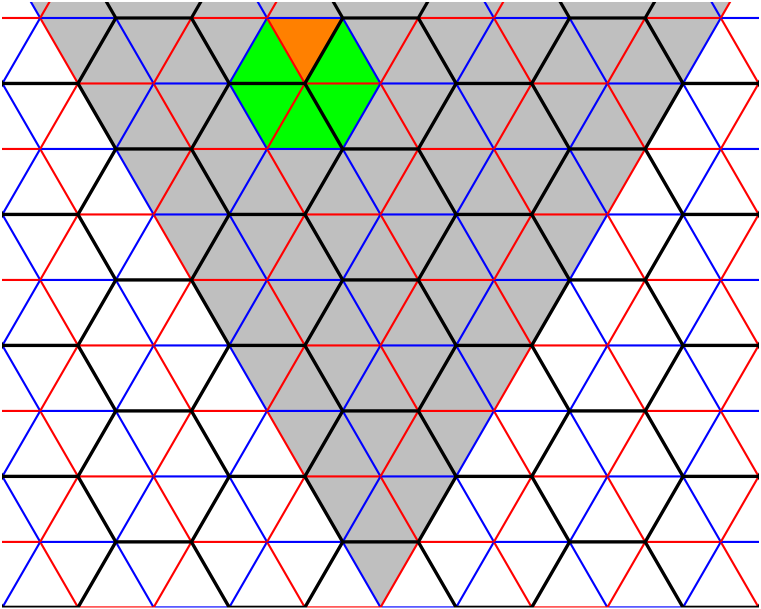

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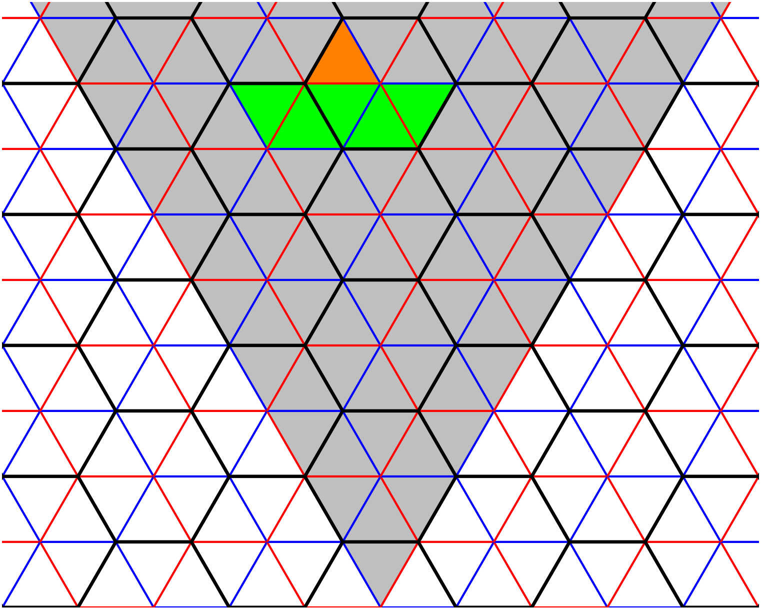

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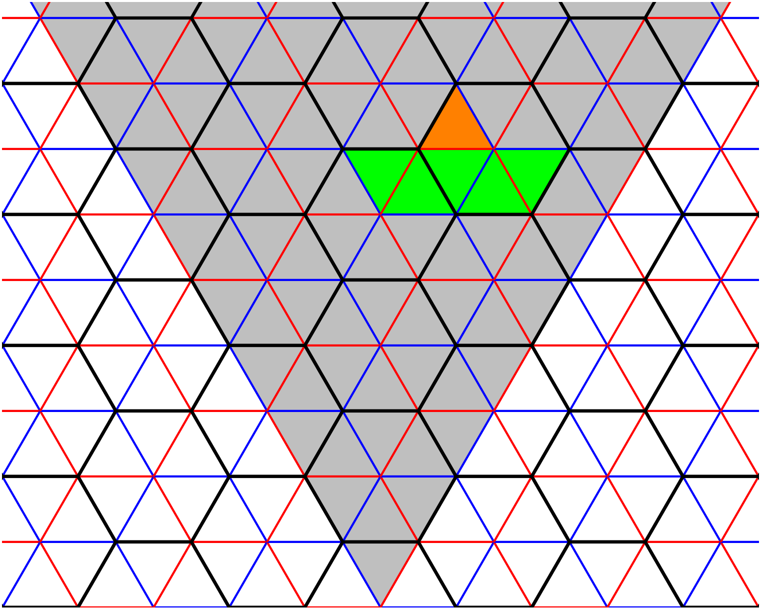

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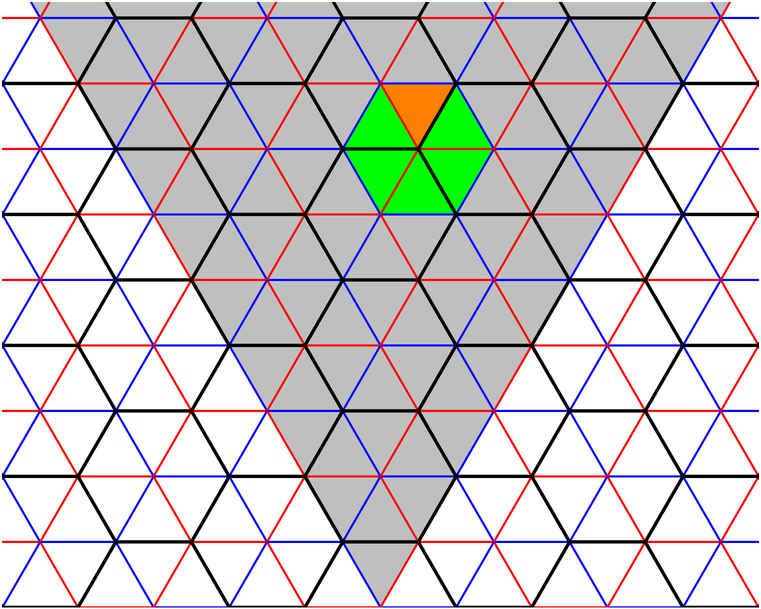

Questions