Solutions to Problems on Implication

Problem 1:

A mother and her son were in the New River Valley Mall. The son was begging for Ice Cream. The mother said to her son,

" If you do not behave, I will not buy you any ice cream!"

Let B be the statement "Behave" and I be the statement "buy ice cream"

Then we have $\sim B \rightarrow \sim I \equiv I \rightarrow B$ where I is sufficient condition and B is the necessary. (It is usually easier to represent statements as positive)

A) Which of the following statements are equivalent to the mother's statement to her son? Justify your answer.

      1) Behave or no ice cream.

Ans: $B \vee \sim I \equiv \sim I \vee B \equiv I \rightarrow B$ and the sufficient and necessary condition match with the original statement.

      2) You must behave before I will buy you any ice cream.

Ans: The word "must" means that B is a necessary condition and I is the sufficient condition which match with the original statement.

      3) Your good behavior guarantees some ice cream.

Ans: The word "guarantees" means that B is the sufficient condition and I is the necessary condition. $B \rightarrow I$ **This does not match the original**

      4) No ice cream indicates that you are not behaving.

Ans: $\sim I \rightarrow \sim B \equiv B \rightarrow I$ Therefore not equivalent to the original statement

      5) If I do not buy ice cream then you are not behaving.

Ans: $\sim I \rightarrow \sim B \equiv B \rightarrow I$ Therefore not equivalent to the original statement

      6) If you have ice cream then you are behaving.

Ans: $I \rightarrow B$ and is equivalent to the original statement

      7) To have ice cream, good behavior is necessary.

Ans: The necessary condition in this sentence is B, so $I \rightarrow B$ and is equivalent to the original statement

      8) Behaving well is sufficient for me to buy you ice cream.

Ans: The sufficient condition is B so $B \rightarrow I$, and is not equivalent to the origianal

      9) Since you are not behaving, I am not buying ice cream.

Ans: $\sim B \rightarrow \sim I \equiv I \rightarrow B$ and is equivalent to the original statement.

      10) Only if I am buying ice cream are you behaving.

Ans: The "Only if" means that I is the necessary condition and $B \rightarrow I$, therefore this statement is not equivalent to the original.

11) If I am buying ice cream then you are behaving.

Ans: $I \rightarrow B$ and is therefore equivalent to the original statement.

B) 1) If you behave and I do not buy any ice cream have I broken a promise?
NO! Because B is the necessary condition.

2) If the son has ice cream did he behave?
Yes, since Ice cream is the sufficient condition.

Problem 2:   Given the following statements:
P: Tech is #1 in the ACC.
Q: UVA always acts appropriately at all ball games.
R: The Marching Virginias are not good.
W: Tech's original colors were black and blue.

The statement $[(P \wedge Q) \vee \sim W] \rightarrow R$ is false and ~Q is true. Determine the truth value of P, Q, R and W. Explain in complete sentences you reasoning for each answer.

Answer: Since ~Q is true then Q is false. The implication is given to be false which means that the statement $[(P \wedge Q) \vee \sim W]$ must be true and R is false. Since Q is false then $(P \wedge Q)$ is false (P can not be determined) and $\sim W$ is true. This means that W is false.

Problem 3: Use the definition of Nand Operator to prove the following:
1) ~P = P|P   2) P^Q = (P|Q)|(P|Q)

Answers: 1) $\sim P \equiv \sim (P \wedge P) \equiv P|P$

2) $P \wedge Q \equiv \sim \sim (P \wedge Q) \equiv \sim (P|Q) \equiv (P|Q)|(P|Q)$