Math 2534 Test 2A Spring 2011 Name______________________________

Show and justify all work. No electronic devices. Pledge: I have neither given nor received help on this test. Signature:____

Problem 1: Using PMI prove the following theorem. Justify each step. Indicate where you use the Inductive assumption.

Theorem: For all real numbers n > 3, $n+3<n!$

Problem 2: Use **set Algebra** to prove the following. Justify each step.

Theorem: For any sets A and B,

$[(A\cup B)\cap A]^c - B^c = B - A$

Problem 3: Use **proof by elements** to prove the following. Justify each step

Theorem: For any sets A and B,

$P(A-B)\subseteq P(A)-P(B)$

Problem 4: Using PMI prove the following theorem. Justify each step. Indicate where you use the Inductive assumption.

Theorem: If $a_1 = a_2 = 1,\ a_n = 2a_{n-1} + a_{n-2}\ n > 2,$ then $a_n < 6a_{n-2},\ n \geq 5$ and $n \in \mathrm{N}$

Problem 5: Consider the set A of all divisors of 24, A = { 1, 2, 3, 4, 6, 8, 12, 24}, with the operations defined as follows: a♥b = LCM(a, b), a♦b = GCD(a,b) and the complement (or negation) is defined to be $a^{/} = 24/a$. Show that the following properties of a Boolean Algebra are valid for arbitrary examples.

1) Is the following property true for 3♥(4♦8) = (3♥4)♦(3♥8) ?
2) Find the identities for each of the operations and justify your choice with an example.
3) Give an example that indicates that DeMorgan's will work.
4) Evaluate 6♥($6^{/}$♦8) = ?.

Problem 6: Let the set A = {a, b, {b}, {a, b}}, set B = {b}, set C = {a, b}, D = {a, {a}}

Find the following:

a) $A \cap D$

b) A – C

c) $B \oplus D$

d) $B \times C$

e) P(D)