Math 2534 Solutions Test 1B Fall (green test)

Problem 1:(18pts) Use Algebra of Logic to simplify the following and justify each step.

Theorem: $[\sim(p \wedge \sim q) \wedge q] \to \sim p \equiv q \to \sim p$

Proof:

$[\sim(p \wedge \sim q) \wedge q] \to \sim p \equiv$	Given
$\sim[\sim(p \wedge \sim q) \wedge q] \vee \sim p \equiv$	Implication Law
$[\sim\sim(p \wedge \sim q) \vee \sim q] \vee \sim p \equiv$	DeMorgan's Law
$[(p \wedge \sim q) \vee \sim q] \vee \sim p \equiv$	Double negative Law
$[\sim q] \vee \sim p \equiv$	Absorption Law
$q \to \ \sim p \equiv$	Implication Law

$\therefore [\sim(p \wedge \sim q) \wedge q] \to \sim p \equiv q \to \sim p$

Problem 2: (18pts) Prove the following: You may use theorems we have proved about prime numbers BUT NOT theorems about even and odd numbers.
(Use only definitions.)

Theorem: The product of two prime integers each greater than 2 is always odd.
Proof:
Given two prime integers a and b greater than 2, by a previous theorem we know that any prime integer greater than two will be odd. Since the prime numbers a and b are odd, we can represent them as follows by the definition of odd. Let a = 2k + 1 and b = 2p + 1 for integers k and p. Now consider the product :

(a)(b) = (2k + 1)(2p + 1) = 4kp +2k + 2p + 1 = 2(2kp + k + p) + 1 = 2m + 1 where
m = 2kp + k + p is an integer.
So the product is indeed odd by definition of odd.

Problem 3: (18pts) Use method of **Contradiction** to prove the following theorem. Your write up needs to be clear concise and well documented with a good conclusion. **Use definitions only.**

Theorem: If n is a natural number, if x^2 is even then $x+5$ is odd.

Proof by contradiction:

Assume that there exist a natural number so that x^2 is even AND $x+5$ is even.

Since we have assumed that x + 5 is even, by definition of even we have x + 5 = 2m for some integer m. Solving for x, we have that x = 2m – 5. Now consider x^2.

$x^2 = (2m-5)^2 = 4m^2 - 20m + 25 = 2(2m^2 - 10m + 12) + 1 = 2p + 1$

where p = $2m^2 - 10m + 12$ is an integer.

So by definition of odd x^2 is odd and this is **a contradiction to the sufficient condition** that x^2 is even. Therefore x + 5 must be odd.

Problem 4: (12pts) Given the following true statements:

1) John is smart.
2) John or Mary is 20 years old.
3) If Mary is 20 years old, then John is not smart.

Put the above statements into symbolic logic and define all variables used.

Determine if the following statement is true or false. Justify your conclusion in a clear presentation. **John is not smart or Mary is not 20 years.**

Let S be John is smart

Let M be Mary is 20 years old.

Let J be that John is 20 years old

Statement 1)	S	John is smart
2)	$J \vee M$	John or Mary is 20 years old.
3)	$M \to\ \sim S$	If Mary is 20 years old, then John is not smart.

The statement **John is not smart or Mary is not 20 years.** $\sim S \vee \sim M \equiv S \to \sim M$

Notice that $\sim S \vee \sim M \equiv S \to \sim M \equiv M \to \sim S$ **by the contrapositive.**

So the statement "John is not smart or Mary is not 20 years" is equivalent to " If Mary is 20 years old, then John is not smart" by the contrapositive and has the same truth value which is T. f

Problem 5: (8pts)

Convert the following into a natural conversational English sentence.

Let x be an element in the domain D of all student clubs
Let y be an element in the domain C of all students
The predicate P(x,y) = y joins x

$\exists y \in C \middle| \forall x \in D, P(x, y)$

Solution: There exist at least one student that joins all student club.

Problem 6: (18pts) Use method of **Contrapositive** to prove the following theorem. Your write up needs to be clear, concise, and well documented with a good conclusion **using definition only.**

Theorem: If the sum of a + b is irrational then a or b is irrational.

Proof by contrapositive:

If a and b is rational, Then a + b is rational.

Since a and b are rational, then by definition of rational $a = \frac{c}{d}$ and $b = \frac{e}{f}$ where c,d,e,f are nonzero integers.

Now consider $a + b = \frac{c}{d} + \frac{e}{f} = \frac{cf + ed}{df} = \frac{m}{n}$ where $m = cf + ed$ and $n = df$ and each are integers. By definition a + b is rational. Since the contrapositive is true the original equivalent statement is also true.

Problem 7: (8pts) Use the Division Algorithm /Quotient Remainder Theorem to answer the following problem:

The integers can be portioned into five distinct groups using $Z \bmod 5$. State the five groups and determine which group would contain the integer n = 52.

Given Z mod 5, by the QRT any integer n can be represented as follows:

n = 5q for some integer q.
n = 5q + 1 for some integer q.
n = 5q + 2 for some integer q.
n = 5q + 3 for some integer q.
OR
n = 5q + 4 for some integer q.

By the Quotient Remainder Theorem we have 52 = 5(10) + 2 where q = 10 and a remainder of 2.