Math 2534 Solutions Test 1 Spring 2018 Name

Problem 1: Determine which of the following are equivalent statements by putting each statement into symbolic **implication** notation and justifying all conclusions by comparing the sufficient and necessary conditions. (identify any logic properties used)
(14pts)
Define Variables as follows: Let P be the statement "you do all the problems"
Let C be the statement "you get at least a C"

a) Only if you do all the problems will you get at least a C in this course.

$C \to P$

b) Not having done the problems guarantees that you get at most a D.

$\sim P \to \sim C \equiv C \to P$ by the contrapositive

c) Do not do the problems or get at least a C.

$\sim P \vee C \equiv P \to C$ by the implication law

Conclusion is: Part a) and b) are equivalent since The sufficient and necessary conditions are the same.

Problem 2: (10pts) Put each statement below into a "standard" English sentence
Let **P** be the set of all polynomials where x is an arbitrary polynomial.
Let **R** be the set of all real roots where y is an arbitrary real root.
Predicate **H(x,y) = x has a y**

1) $\forall x \in P, \exists y \in R | H(x, y)$

Every polynomial has a real root.

2) $\exists y \in R | \forall x \in P, \sim H(x, y)$

The is a real root that does not satisfy any polynomial.

Problem 3: (14pts)
Use Algebra of Logic to simplify the following and justify each step with the appropriate logic property. Present your work in proper theorem-proof form.

Theorem: For all propositions p and q, $[\sim p \wedge \sim (q \wedge p)] \rightarrow [q \wedge (p \vee \sim q)] \equiv p$
$[\neg p \wedge \neg(q \wedge p)] \rightarrow [q \wedge (p \vee \sim q)] \equiv p$

Proof:
$[\sim p \wedge \sim (q \wedge p)] \rightarrow [q \wedge (p \vee \sim q)] \equiv$
$\sim [\sim p \wedge \sim (q \wedge p)] \vee [q \wedge (p \vee \sim q)] \equiv$ by the Implication Law
$[\sim\sim p \wedge \sim\sim (q \wedge p)] \vee [q \wedge (p \vee \sim q)] \equiv$ by the DeMorgan's Law
$[p \wedge (q \wedge p)] \vee [q \wedge (p \vee \sim q)] \equiv$ by the Double Negative Law
$[p] \vee [q \wedge (p \vee \sim q)] \equiv$ by the Absorption Law
$p \vee [(q \wedge p) \vee (q \wedge \sim q)] \equiv$ by the Distributive Law
$p \vee [(q \wedge p) \vee (F)] \equiv$ by the Inverse Law
$p \vee (q \wedge p) \equiv$ by the Identity Law
p by the Absorption Law

Therefore $[\sim p \wedge \sim (q \wedge p)] \rightarrow [q \wedge (p \vee \sim q)] \equiv p$

Problem 4: (12pts)
Using a **direct** method to prove the following **using definitions only (state them correctly.)**
Theorem: Let a be an integer. If $2 \mid (a-1)$, then a is odd.

Proof: Given that a be an integer and $2 \mid (a-1)$, by definition of divisible, there exist an integer k so that $2k = a - 1$. Now solve for a to get $2k + 1 = a$. But this satisfies the definition of odd since k is an integer and we have that a is odd.

Problem 5: Prove the following by method of contrapositive (use definitions only).
Theorem: If 3p + 2 is irrational then p is also irrational.
(14pts)
Proof by Contrapositive: The restatement is given to be If p is rational then 3p + 2 is rational.

Given that p is rational, by definition of rational, there exists integers a, b with b not zero so that

$p = \frac{a}{b}$. Now consider $3p + 2 = 3\left(\frac{a}{b}\right) + 2 = \frac{3a + 2b}{b} = \frac{m}{n}$ where m = 3a + 2b is an integer.

Therefore 3p + 2 = m/n is rational by definition of rational.

Since the contrapositive is true, the original equivalent statement is also true and

if 3p + 2 is irrational then p is also irrational

Problem 6: (12pts) Put the following argument into symbolic **implication** logic and determine the conclusion using the transitive argument form. Use sentences to explain your reasoning and justify your conclusion with the appropriate logic properties. Define your variables in sentences.

Argument: Only if Ed goes will Bill also go. Ed will not go if Jessica goes. Bill's sister does not go or Jessica will go. Therefore______________________________

Let B be the statement: "Bill goes."

Let E be the statement: "Ed goes."

Let J be the statement: "Jessica goes."

Let S be the statement: " Bill's Sister goes."

Symbolic Logic:

$B \to E \equiv \ \sim E \to \sim B$ by contrapositive law
$J \to \sim E$
$\sim S \vee J \equiv S \to J$ by implication law

Using the transitive argument form we have that $S \to J \to \sim E \to \sim B$, so $S \to \sim B$
Therefore the conclusion is If Bill's sister goes, then Bill will not.

Problem 7: (14pts) Prove the following theorem using previous theorems ONLY. Choose the theorems
needed from the theorems listed below and **quote** the theorems used **in their entirety.**
THEOREM: If k is a prime integer, then $k^2 - k(k-1)$ is odd.

Resource theorems:

a) The sum or difference of an even and odd integer is odd.
b) The product of two odd integers is odd.
c) The product of an even integer and any other integer is even.
d) The sum or difference of two odd integers is even
e) Any prime number n > 2, is odd.
f) Any two consecutive integers have opposite parity.
g) The product of two consecutive integers is even.

If k is a prime greater than 2, then k is prime since any prime greater than 2 is odd. Given that k is odd, k^2 is also odd, since $k^2 = (k)(k)$ and the product of any two odd integers is odd. Notice that k and k – 1 are consecutive integers and therefore have opposite parity. The product k(k – 1) is even since the product of two consecutive integers is always even. The difference, $k^2 - k(k-1)$, is odd since the difference of an odd and even integer is always odd.

Problem 8: (10pts) Given that $(B \to C) \to (A \vee D)$ is false and $\sim B$ is false, find the truth value for the statement $(\sim C \vee A) \to (B \wedge D)$. Justify all your conclusions in sentences referring to the definitions of $\wedge, \vee, \to$ and their truth values. (Do Not show a full truth table to determine the results)

Given that $\sim B$ is false, we know that B is true. Since $(B \to C) \to (A \vee D)$ is false the sufficient condition $(B \to C)$ must be true and the necessary conclusion $(A \vee D)$ must be false. Given that B is true, in order for the implication $B \to C$ to be true, C must also be true. The statement $A \vee D$ is false so that A and D must both be false. To summarize B, C are true and A,D are false.

Now consider $(\sim C \vee A) \to (B \wedge D) \equiv (\sim T \vee F) \to (T \wedge F) \equiv (F \vee F) \to (T \wedge F) \equiv F \to F = T$