Math 2534 Solutions Homework 9 Spring 2018 Show all work and staple multiple sheets.

Problem 1: Use set containment to derive a conclusion from the following statements (A Lewis Carroll puzzle)

No one takes in the Times, unless he is well educated.
No hedgehogs can read.
Those who cannot read are not well educated.

Solution: Let T be the set of everyone who takes the times. Let E be the set of everyone who is well educated. Let H be the set of hedgehogs. Let R be the set of everyone who can read.

Statement 1: $T \subset E$, **Statement 2:** $H \subset R^C$, **Statement 3:** $R^C \subset E^C$

By theorem: $IA \subset B \to B^C \subset A^C$, we have that $T \subset E \to T^C \subset E^C$

So by containment we have that $H \subset R^C \subset E^C \subset T^C$, Therefore hedgehogs do not take the Times.

Problem 2:
If $A \times B = \{(a,b),(b,b),(c,b),(a,a),(b,a),(c,a)\}$ then find the elements in each A and B.

Set A = {a, b, c} and Set B ={b, a}

Problem 3: Find the power set for :

a) B = { a, b, c, d}

$P(B) = \{\varnothing, \{a,b,c,d\}, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\},$
$\{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$

b) A = {∅, {∅}}

$P(A) = \{\varnothing, \{\varnothing, \{\varnothing\}\}, \{\varnothing\}, \{\{\varnothing\}\}\}$

Problem 4: Given sets $A = \{a, b, \{c\}, c\}, B = \{a, \{b, c\}, d, \varnothing\}, C = \{b, c\}$

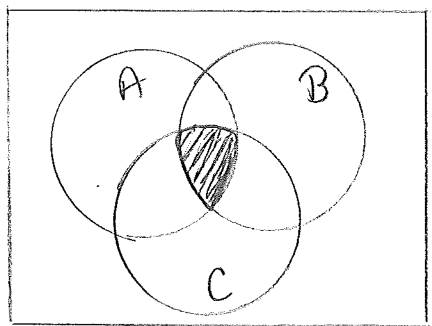
Find the following: (don't forget to use equal signs.)

a) Find the following sets:

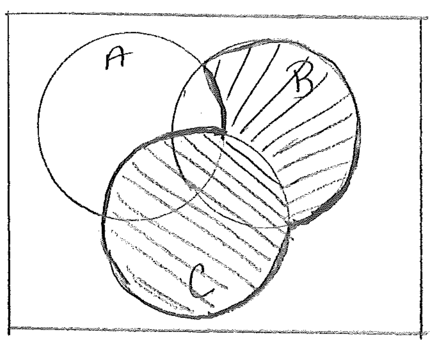
1) $A \cap B = \{a\}$
2) $B \cup C = \{a, \{b, c\}, d, \varnothing, b, c\}$
3) $B - A = \{\{b, c\}, d, \varnothing\}$
4) $B \cap C = \varnothing$
5) $A - C = \{a, \{c\}\}$

Problem 5: Draw a Venn diagram for each of the following sets.

a) $A - (B \cap C)^C$



b) $(A^C \cap B) \cup C$



Problem 6: Using proof by elements, prove the following:

Theorem 1: For all sets A, B, C, D, if $A \subseteq C$ and $B \subseteq D$, then $A \cap B \subseteq C \cap D$

Proof:

$\forall x, x \in A \cap B \rightarrow x \in A \wedge x \in B$	by definition of intersection
$\rightarrow x \in C \wedge x \in B$	Since $A \subseteq C$
$\rightarrow x \in C \wedge x \in D$	Since $B \subseteq D$
$\rightarrow x \in C \cap D$	by definition of intersection
Therefore $A \cap B \subseteq C \cap D$	by definition of containment

Theorem 2: For all sets A, B, If $(A \cup B)^C = A^C \cap B^C$
Proof:

$\forall x, x \in (A \cup B)^C \rightarrow x \notin A \cup B$	by definition of complement
$\rightarrow \sim (x \in A \cup B)$	by definition of negative
$\rightarrow \sim (x \in A \vee x \in B)$	by definition of union
$\rightarrow \sim (x \in A) \wedge \sim (x \in B)$	by DeMorgan's Law
$\rightarrow (x \notin A) \wedge (x \notin B)$	by definition of negative
$\rightarrow (x \in A^C) \wedge (x \in B^C)$	by definition of complement
$\rightarrow x \in A^C \cap B^C$	by definition of union

So we have that $(A \cup B)^C \subseteq A^C \cap B^C$ by definition of containment.
By reverse steps we can show that $A^C \cap B^C \subseteq (A \cup B)^C$.
Therefore $A^C \cup B^C = (A \cup B)^C$ by the definition of Equal sets.

Theorem 3: For all sets A, B, If $A \subseteq B$ Then $\mathrm{A} \cap \mathrm{B}^C = \varnothing$ (Use proof by contradiction.)
Proof:
Assume that $\mathrm{A} \cap \mathrm{B}^C \neq \varnothing$ then there exist an element x in $\mathrm{A} \cap \mathrm{B}^C$. So there is an element in A that is not in B and by definition $A \not\subset B$. But this is a contradiction since it is given that $A \subseteq B$. Therefore $\mathrm{A} \cap \mathrm{B}^C = \varnothing$.

Problem 7: Prove using Set Algebra:
For all sets A,B, $[A-(B \cap A^C)]^C \cap B = B - A$

$[A-(B \cap A^C)]^C \cap B =$	given
$[A \cap (B \cap A^C)^C]^C \cap B =$	by difference law
$[A^C \cup (B \cap A^C)] \cap B =$	by DeMorgan's law
$A^C \cap B =$	by absorption law
$B \cap A^C =$	by commutative law
$B - A =$	by difference law

$\therefore [A-(B \cap A^C)]^C \cap B = B - A$