

Math 2534 Solution Homework 1 (sec 2.1) Spring 2018

Problem 1: Given the propositions:

P: Mary is in CS is a true statement

Q: John is an Engineer is a false statement

R: Grace is a Math Major is a true statement

Determine the truth values of the following compound propositions; Show your work in detail using correct notation.

$$1) (P \wedge \sim R) \vee \sim (P \wedge Q) \equiv (T \wedge \sim T) \vee \sim (T \wedge F) \equiv (T \wedge F) \vee \sim (F) \equiv F \vee T \equiv T$$

$$2) (P \oplus Q) \wedge \sim R \equiv (T \oplus F) \wedge \sim T \equiv T \wedge F \equiv F$$

Problem 2:

Let p, q and r be the propositions given below:

p: The birdwatchers are in the woods.

q: A blue heron was seen at the lake.

r: Migration starts soon.

Part A:

Put the sentences below into symbolic logic notation.

a) No blue heron was seen at the lake, but the birdwatchers are in the woods.

$$\sim q \wedge p$$

b) It is not true that migration will not start soon.

$$\sim \sim r \equiv r$$

c) Migration starts soon or a blue heron is seen at the lake.

$$r \vee p$$

d) Either the birdwatchers are in the woods or migration has not started soon.

$$p \oplus \sim r$$

Part B:

Put the symbolic logic notation into English sentences.

$$a) (p \wedge q) \vee \sim p$$

The bird watchers are in the woods and the blue heron was seen at the Lake, or the bird watchers are not in the woods.

$$b) \sim (p \vee r) \wedge q$$

It is not true that the bird watchers are in the woods or that the migration starts soon, and the blue heron was seen at the lake.

c) $(p \oplus \sim q) \wedge \sim q$ Either the bird watchers are in the woods or the blue heron was not seen at the lake but not both and the blue heron was not seen at the lake.

Problem 3: For each of the sentences below, determine whether an ‘exclusive or’ or an ‘inclusive or’ is intended.

1) To go the game cheerleaders can go either by bus with the band or by air with the team.

Exclusive OR

2) The next question is true or false. **Exclusive OR**

3) You can help clean the kitchen or the bathroom. **Inclusive Or**

4) You may either go to the retirement party or the play which are each at 8pm tonight.

Exclusive OR

5) The Girl Scouts can sell cookies door to door or hold a cookie booth at Kroger’s.

Inclusive OR

Problem 4:

1) Construct a truth table to verify the following equivalence relation:

$$\sim (p \wedge \sim q) \vee \sim q \equiv \sim p$$

p	q	$\sim q$	$p \wedge \sim q$	$\sim (p \wedge \sim q)$	$\sim (p \wedge \sim q) \vee \sim q$	$\sim p$
T	T	F	F	T	T	F
T	F	T	T	F	F	F
F	T	F	F	T	T	T
F	F	T	F	T	F	T

Notice that columns 6 and 7 do not have the same truth values, Therefore

$\sim (p \wedge \sim q) \vee \sim q \equiv \sim p$ is not valid

2) Construct a truth table to verify the following equivalence relation:

$$\text{De Morgan's Law } \sim (p \vee q) \equiv \sim p \wedge \sim q$$

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$p \vee q$	$\sim (p \vee q)$
T	T	F	F	F	T	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	T	F	T

Notice that columns 5 and 7 have the same truth values, Therefore

$$\sim (p \vee q) \equiv \sim p \wedge \sim q$$

Problem 5:

Logic Puzzle:

Solve the following puzzle and then write up a paragraph explaining your reasoning. Use your own judgment on how to write this up. It needs to be clear to anyone who tries to follow your write up. Do not use tables or charts in your presentation.

The Spelling contest:

There were five finalist at the Blacksburg Middle School Spelling Bee. They competed till four of the five had misspelled a word and the last student was declared the winner. Given the following statements below (one of which is false), find the first and last name of each of the finalist where surnames are given to be Snider, Johnson, Owens, Keener, or Murphy. Determine who missed which word and what was the order in which they finished the competition.

STATEMENTS:

- 1) Owens, who was not the one who misspelled “pneumonia” had not participated in a Spelling Bee before He said he had not expected to do well.
- 2) “Vicissitude” was misspelled immediately after “bivouac” and immediately before “isthmus”.
- 3) Eleanor was happy that her good friend Snider was one of the two finalist in the contest.
- 4) Before Eric misspelled his word there were only two contestants left. He and Keener had studied together to prepare for the Spelling Bee.
- 5) Max, who lasted longer in the competition than Snider was not the winner.
- 6) Johnson, who is not Anna, was the first to misspell a word.
- 7) Anna, Snider, and Keener were neighbors.
- 8) Snider told Louise that she could not sleep the night before the contest.

Solution:

Anna’s last name is Murphy. We can determine this from statements 6, 7 and 1 which state that Anna is not Johnson, Snider, Keener or Owens (since Owens is not a women) Eric’s last name is Owens. We can determine this from statements 4, 6 and 8. Since Eric came in second he is not Johnson nor is he Keener. We know he is not Murphy and Snider is a woman.

Eleanor’s last name is Snider. We can determine this from statements 5 and 8 which states that neither Max or Louise is Snider.

Max’s last name is keener. We can determine this from statements 5 and 6. Johnson was the first to misspell a word and Max lasted longer therefore Max is not Johnson. This means that Louise must be Johnson by elimination.

We now know that Lousie Johnson was the first to misspell a word (statement 6) so she is In 5th place. Eric Owens was in 2nd place by statement 4.

Eleanor Snider was in 4th place and Max Keener was 3rd place by statement 5.

Therefore by elimination, the first place winner was Anna Murphy.

By statements 1 and 2 we know “physiognomy” was not the word Eric Owens missed spelled and by statement 2 we know that “bivouac” , “vicissitude” and “isthmus” were

misspelled in that order. So “physiognomy” must be the first word misspelled and Eric misspelled “isthmus”.

To sum up the results, we have the following: Statement 3 was found to be false since contradicted that logically Snider could not be one the two finalists.

Anna Murphy was the first place winner with no misspelled words.

Eric Owens was in second place having missed “isthmus”.

Max Keener was in third place having missed “vicissitude”.

Eleanor Snider was in fourth place having missed “bivouac”.

Louise Johnson was fifth place having missed “pneumonia”.