<u>Math 2534 Solution Homework 10 Spring 2018</u>

Show all work and staple multiple sheets.

Problem 1:

Use Proof By Set Algebra to prove the following where A, B are Sets
Justify each step. Convert to union and intersection operations only.

$[A^C \cup (B - A)]^C - A^C = A$

Theorem: For any sets A, B $[A^C \cup (B - A)]^C - A^C = A$

Proof:

$[A^C \cup (B - A)]^C - A^C =$	Given
$[A^C \cup (B \cap A^C)]^C \cap A^{CC} =$	Difference Law
$[A^C \cup (B \cap A^C)]^C \cap A =$	Double Complement Law
$(A^C)^C \cap A =$	Absorption Law
$A \cap A =$	Double Complement Law
A	Idempotent Law

$\therefore \quad [A^C \cup (B - A)]^C - A^C = A$

Problem 2: Define a Boolean Algebra as follows:

Let B = { 1, 2, 3, 5, 6, 10, 15, 30} be the set of all positive divisors of 30 with operations defined on the set to be as follows: a + b = LCM(a,b) , and the a • b = GCD(a, b)

The complement is defined $\bar{a} = \frac{30}{a}$

a) Determine the identities for each operation.

The identity for "+" is 1 and the identity of "•" is 30.

b) Evaluate $5 + \bar{5}$ and $3 \bullet \bar{3}$ and determine if the results are correct for a Boolean algebra.

We have that $5 + \bar{5}$ = 5 + 30/5 = 5 + 6 = 30, $3 \bullet \bar{3} = 3 \bullet (30/3) = 3 \bullet 10 = 1$

c) Determine if DeMorgan's Law is valid for the following: $\overline{(6+15)} = \bar{6} \bullet \overline{15}$

$\overline{(6+15)} = \overline{30} = 30/30 = 1$

$\bar{6} \bullet \overline{15} = (30/6) \bullet (30/15) = 5 \bullet 2 = 1$

$\therefore \overline{(6+15)} = \bar{6} \bullet \overline{15}$

Problem 3: Fill in the reasons for each step for the following.

Theorem:

For all a in the Boolean Algebra B under the operations of $+, \bullet$, $(a+\bar{a})+\overline{(h\bullet a)}=k$, where k is the identity for operation $\bullet$ and h is the identity for +.

Proof:

$(a+\bar{a})+\overline{(h\bullet a)}=$ ____Given________

$k+\overline{(h\bullet a)}=$ _______Complement law__________

$k+(\bar{h}+\bar{a})$ ________DeMorgan's law_________

$k+(k+\bar{a})=$ ________Identity complements_________

$k+k=$ _______Universal bound law__________

k _______Idempotent law__________

Problem 4:

Let a and b be elements in the Boolean Algebra B with the following defined operations. Operation 1 is $\otimes$ with identity $\boldsymbol{p}$ and operation 2 is $\odot$ with identity $\boldsymbol{q}$. The complement (or negative) is represented by $\bar{a}$. Prove the following:

Theorem: For any elements a, b in B $\bar{a}\otimes[(\overline{a\otimes b})\otimes(b\odot\bar{q})]=\bar{a}$

Proof:

$\bar{a}\otimes[(\overline{a\otimes b})\otimes(b\odot\bar{q})]=$ Given

$\bar{a}\otimes[(\bar{a}\odot\bar{b})\otimes(b\odot\bar{q})]=$ DeMorgan's Law

$\bar{a}\otimes[(\bar{a}\odot\bar{b})\otimes(b\odot p)]=$ Identity Complement Law

$\bar{a}\otimes[(\bar{a}\odot\bar{b})\otimes p]=$ Universal Bound Law

$\bar{a}\otimes[(\bar{a}\odot\bar{b})]=$ Identity for $\otimes$ Law

$\bar{a}$ Absorption Law

$\therefore \bar{a}\otimes[(\overline{a\otimes b})\otimes(b\odot\bar{q})]=\bar{a}$