Math 2534: Lecture Sheet on Methods of Proofs:

Outline for proof process:

1) Always write a clear statement of the conjecture which must explicitly or implicitly use the universal quantifier.
2) All notation and variables must be clearly defined.
3) (Be sure that all the definitions that you will use is stated or already known to your readers)
4) Body of proof: Every step in the proof must be justified. Always use complete and clear sentences.
5) There needs to be a conclusion that summarizes the proof and makes clear what has been demonstrated.

Proof Methods:

a) Direct $\forall x \varepsilon D, P(x) \rightarrow Q(x)$

b) Indirect:

1) Contrapositive $\forall x \varepsilon D, \sim Q(x) \rightarrow \sim P(x)$

2) Contradiction $\sim [\forall x \varepsilon D, P(x) \rightarrow Q(x)] \equiv \exists x \varepsilon D, P(x) \wedge \sim Q(x))$

c) Disproof by counter example $\exists x \varepsilon D | \; P(x) \wedge \sim Q(x)$

Common mistakes in writing up a proof:

1) Using the same variable notation for two different things.
2) Jumping to the conclusion without showing all deductive steps.
3) BEGGING THE QUESTION; This means you are assuming and Using what you are trying to prove.
4) Misuse of the word "IF". It can reflect imprecise thinking that leads to trouble later in the proof.
5) Not using complete English sentences to help tie deductive steps together.

Below are Definitions that we will use:
You must be precise in using these definitions

1) Definition of an even integer:

n $\in$ Z is even if and only if $\exists$ k $\in$ Z $\ni$ n = 2k

2) Definition of an odd integer:

n $\in$ Z is odd if and only if $\exists$ k $\in$ Z $\ni$ n = 2k + 1

3) Definition of a prime integer:

$n \in Z$ is prime iff for $n > 1, \forall r,s \; \varepsilon \; Z^+$, if $n = (r)(s)$, then $r = 1 \vee s = 1$

4) Definition of a composite integer:

$n \in Z$ is composite iff $\exists \, r,s \; \varepsilon \; Z^+ \ni n = (r)(s) \wedge r \neq 1 \wedge s \neq 1$.

5) Definition of a rational number:

r is rational iff $\exists \, a,b \; \varepsilon \; Z, \ni r = \frac{a}{b} \wedge b \neq 0$

6) Definition of Divisible;

$d|n$ if and only if $\exists \, k \; \varepsilon \; Z \ni n = dk$.

(this means that d divides n and has zero remainder)

7) Definition of Consecutive integers:

Two integers are consecutive iff one is one more than the other.