Math 2534 Resource Sheet for Set Theory Definitions.

Definitions:
If a is a member of a set A, it is called an element of A ($a \in A$)
The null set, $\varnothing$, has no members.
The universal U has all elements under consideration as members.
Order of members in a set does not matter. A member is mentioned only once.

Definition of How Sets Relate to each other. (subsets and containment)
A is properly contained in B, $A \subset B \text{ iff } \forall x \in U, x \in A \to x \in B \wedge (\exists y \in B \wedge y \notin A)$
A is contained in B, $A \subseteq B \text{ iff } \forall x \in U, x \in A \to x \in B$
A is not contained in B, $A \not\subset B \text{ iff } \exists x \in U \mid x \in A \wedge x \notin B$
$A = B, \text{ iff } \forall x \in U, (x \in A \to x \in B) \wedge (x \in B \to x \in A)$, **ie.** $A = B \text{ iff } (A \subset B) \wedge (B \subset A)$
$\varnothing$ is a subset of every other set
Every set is a subset of itself.

Defined Operations on the Set of all Sets
The operation of Union is represented by $\bigcup$
The operation of Intersection is represented by $\bigcap$
The complement of A is A^C where $A \cap A^C = \varnothing$

$A \cup B \text{ iff } x \in U \mid x \in A \vee x \in B$
$A \cap B \text{ iff } x \in U \mid x \in A \wedge x \in B$
Difference $A - B \text{ iff } x \in U \mid x \in A \wedge x \notin B$
$A^C \text{ iff } x \in U \mid x \notin A$
Symmetric Difference $A \oplus B \text{ iff } x \in U \mid x \in A - B \vee x \in B - A$
$A \oplus B = (A - B) \cup (B - A)$
$A \cap B = \varnothing$ means disjoint sets. No members in common

See definitions of Power sets and Cartesian Products in sec 6.1
The next step is develop and prove properties of sets found on page 355 in the text book.