Math 2534 Test 3 Solution Spring 2018

Problem 1: Given the set A = { a, b, c, d}, define an equivalence relation R on the set A, by incorporating the **minimal number** of elements needed when R contains the elements (a, c), (c, d) and (b,b).

Solution (12 pts)

R = { (a,c),(c,d),(b,b), (c,a),(d,c),(d,d),(a,a),(c,c),(d,a),(a,d)}

Problem 2; Let C and D be disjoint subsets of set A so that $A = C \cup D$ and $f : C \to B$ and $g : D \to B$. Define a function h(x) as follows: $h(x) = \begin{cases} f(x) \text{ if x} \in \text{C} \\ g(x) \text{ if x} \in \text{D} \end{cases}$

Determine true or false that **if h is onto then f or g is onto** and justify your conclusion.

Solution (12pts) This statement is false. **See counter example below.**

Let $A = C \cup D, C = \{1,2\}, D = \{3,4\}, B = \{a,b,c,d\}$

Define f and g as follows:

$h(1) = f(1) = a$

$h(2) = f(2) = b$

$h(3) = g(3) = c$

$h(4) = g(4) = d$

Notice that h(x) is onto but f(x) and g(x) are neither an onto function.

Problem 3:

An congruent relation R is defined on the set of integers so that aRb if and only if $a \equiv b \bmod 4$. This relation R partitions the integers into equivalence classes. Verify that [-3] and [-7] are in the same equivalence class. Use an equivalent form of "congruent" to find the correct equivalent class.

Solution (10pts) I will use the equivalent form, $4|a-b$, of the congruent form $a \equiv b \bmod 4$ where a = 0,1,2, or 3. If b = -3 then let a = 1, If b = -7 then let a = 1. This results gives us the following equivalence class, [1] = [-3] = [-7].

Problem 4:
Let a set of elements make up a Boolean Algebra B with operation $\oplus$ with identity **k** and operation $\odot$ with identity **p**. let the complement of a be $\tilde{a}$. Simplify the following expressions and state the appropriate property(s) used.

Solution (16pts)

1) $a \otimes p = p$ by the universal bound law

2) $a \odot a = k$ by the complement law

3) $a \otimes a = a \otimes a = a$ by the double complement law and the idempotent law

4) $(a \odot \tilde{p}) \otimes a = a$ by the absorption law

Problem 5:
Verify that R is an equivalence relation on the integers when xRy iff xy > 0, by showing that R is reflexive, symmetric and transitive.

Solution (18pts)

In order to prove that R is an equivalence relation, R must be reflexive, symmetric and transitive.

To show that R is reflexive, we must show for all integers a (not including zero), aRa. We must verify that $aa > o$ for any integer a. We notice that for all $aa = a^2$ which is always positive. Therefore R is reflexive.

To show that R is symmetric, we must show that for any integers a,b, IF aRb, THEN bRa.
We will assume that aRb which means that ab > 0. Since we are dealing with integers we know that ba = ab > 0 by the commutative law. Therefore R is symmetric.

To show that R is transitive, we must show that for any integers a,b,c, IF aRb and bRc then aRc.
We will assume that aRb and bRc which means that ab > 0 and bc > 0.
In order for the products to be positive, a,b must have the same sign, which is also true for b,c. Since the integer b is in both products, a and c must have the same sign as b. Therefore a, c have the same sign and the product ac > 0 and R is transitive.

Since R is reflexive, symmetric and transitive, R is an equivalence relation.

Problem 6: Prove the following using **method of contradiction.**

Theorem: For non-empty sets A and B, if the function $f : A \to B$ is one to one then the inverse **relation** is well defined.

(18pts)

Proof by Contradiction:

Assume there exist at least one function *f* so that *f* is one to one and its inverse relation, f^{-1}, is not well defined.

We will assume that the relation f^{-1} is not well defined which means that there exist at least one value b in B so that $f^{-1}(b) = a_1$ and $f^{-1}(b) = a_2$, $a_1 \neq a_2$ for $a_1, a_2 \in A$. By definition of inverse relations we have that $f^{-1}(b) = a$ if and only if $f(a) = b$. This means that $f(a_1) = b$ and $f(a_2) = b$. This contradicts that $f(a) = b$ is a one to one function. Therefore the inverse relation must be well defined.

Problem 7:

Given that $f : A \to B,$ and $g : B \to C,$ If $(g \circ f)(2) = (f^{-1} \circ f)(3),$ and $g^{-1}(3) = 6,$ find $f(2)$.

Explain each step of your reasoning.

Solution (14pts)

We are given that $(g \circ f)(2) = (f^{-1} \circ f)(3),$ and $g^{-1}(3) = 6$.

By definition of inverse relations we have that g(6) = 3.

Since the composite of a function and its inverse will be the identity function, we have that $(f^{-1} \circ f)(3) = 3$.

By definition of composite function $(g \circ f)(2) = g(f(2)) = 3 = g(6)$, so $f(2) = 6$.