Math 2534 Homework 5: Proof by Contradiction Spring 2018

Show all work and staple multiple sheets.

Problem 1: Indirect Proof by contradiction -definitions only

Theorem: For all non-zero rational numbers, the product of a rational number and an irrational number is always irrational.

Proof: Contradiction method: Assume that there exist at least one pair of rational and irrational values where the product is rational.

Let ***r*** be a non-zero rational number and ***w*** be an irrational number so that the product ***rw*** is rational. By definition of rational, there exist non-zero integers a, b, c, d so that

$r = \frac{a}{b}$ and $rw = \frac{c}{d}$.

Now consider the product below:

$$rw = \frac{c}{d}$$

$$\frac{a}{b}w = \frac{c}{d}$$

$$w = \left(\frac{c}{d}\right)\left(\frac{b}{a}\right) = \frac{cb}{da} = \frac{m}{n},$$ where m = cb and n = da are each integers

By definition of Rational, $w = \frac{m}{n}$ is a rational number. But this contradicts that w is given to be irrational. Therefore the original statement is true and the product *rw* is irrational.

Problem 2: Indirect Proof by contradiction -definitions only

Theorem: If m and n are integers and the product (m)(n) is even then m is even or n is even.

Proof: Contradiction method: Assume that there exist integers m and n so that the product (m)(n) is even, but m and n are each odd.

If we assume that m and n are each odd then by definition of odd, there exist integer p and q so that m = 2p + 1 and n = 2q + 1.

Now consider the product:

(m)(n) = (2p + 1)(2q + 1) = 4pq + 2p + 2q +1 = 2 (2pq + p + q) + 1 = 2h + 1

where h = 2pq + p +q is an integer. By definition of odd we have that (m)(n) = 2h + 1 must be odd. This contradicts that the product (m)(n) is given to be even. Therefore the original statement must be true and m is even or n is even.

Problem 3: Indirect Proof by contradiction -definitions only
Theorem: For all integers n, $6n+1$ is not divisible by 6.
Proof: Contradiction method:
Assume that there exist at least one integer n and 6 divides 6n + 1 evenly.

Assume that $6|(6n+1)$. By definition of divisible, there exist an integer p so that 6p = 6n + 1. So 6p – 6n = 1 where p and n are known to be integers. So 6(p – n) =1. Let q = p – n which is also an integer and 6q = 1. Solving for q we get that q = 1/6 which is not an integer. Therefore we have a contradiction and 6 does not divide 6n + 1 evenly.