Math 2534 Solution Homework 4 Proof methods Spring 2018

Follow homework requirements to avoid point deductions.

Problem 1:
Determine if the following is true or false and justify your conclusion.

$$\left(\forall x, x \in D_1 \rightarrow [\exists y \,|\, y \in D_2 \wedge P(x,y)]\right) \rightarrow \left(\forall y, y \in D_2 \rightarrow [\exists x \,|\, x \in D_2 \wedge P(x,y)]\right)$$

Solution:
The first statement says that for each arbitrary x-value, there will exist an appropriate y-value so that P(x,y) is true. (But this does not guarantee that all y-values in the D_2 will be used.)
The second statement says that for all y-values there is an appropriate x-value so that P(x,y) is true. However the first statement <u>does not imply</u> the second statement is true since the first statement does not guarantee that all y-values in D_2 will be included.

Problem 2: Use direct proof or counter example. (use previous theorems only)
Theorem: If a and b are odd integers and c is an even integer, then $(a^2 - 1)b + c$ is an even integer.

Proof: Given that *a* is an odd integer, then we know that $a^2 = (a)(a)$ since the product of two odd integers is always odd. We also know that the difference of two odd integers is even, so $a^2 - 1$ is even. Since *b* is also odd and the product of an even and odd integer is even, we have that $(a^2 - 1)b$ is even. The integer *c* is given to be even and the sum of two even integers is even, so $(a^2 - 1)b + c$ is even.

Problem 3: Use direct proof or counter example. (use definitions only)
Theorem: For integers a, b, and c, if a|c, and a|b then a| (b – 2c).

Proof: Given that a|c, and a|b , then by definition of Divisible there exist integers p and q so that c = ap and b = aq. Now consider b – 2c = aq – 2(ap) = a(q – 2p) = am where m = q – 2p is also an integer. Therefore by definition of Divisible, b – 2c = 2m means that a| (b – 2c).

Problem 4: Use direct proof or counter example. (Use definitions only)
Theorem: The quotient of two rational numbers is rational.

Proof: Let r_1 and r_2 be two rational numbers. By definition of rational there exist **non-zero** integers *a, b, c, d* where and $r_1 = \frac{a}{b}$ and $r_2 = \frac{c}{d}$.

Consider the quotient $\frac{r_1}{r_2} = \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \left(\frac{a}{b}\right)\left(\frac{d}{c}\right) = \frac{ad}{bc} = \frac{m}{n}$ where $m = ad$ and $n = bc \neq 0$ are also integers. Therefore by definition of rational the quotient $\frac{r_1}{r_2} = \frac{m}{n}$ is rational.

Problem 5: Prove by direct proof or counter example. (use definitions only)
Theorem: If $a|bc$, then $a|b$ for integers a, b, c

Counter Example: Let a = 8, c = 12 and b = 2, Clearly 8 does not divide 12 or 2 evenly, 8 does divide (12)(2) = 24 evenly.

Problem 6: Use proof by **Contrapositive** or counter example. (use previous theorems only) **Theorem:** If n^3 is odd, then n is odd where n is a natural number.
Proof: by contraposition:
Restatement of Theorem: If n is even then n^3 is even for n $\in$ N.
Given n is even, we know that $n^2 = (n)(n)$ is even, since the product of **two** even integers is always even. This also means that $n^3 = (n^2)(n)$ is even since the product of **two** even integers is even. Therefore n is even $\rightarrow n^3$ is even.
Since the contrapositive is true the equivalent original statement is also true and n^3 is odd, then n is odd.

Problem 7: Use proof by **Contrapositive** or counter example. (use definitions only)
Theorem: For all non-zero integers **a**, **b**, and **c**, if **a** does not divide **bc** then **a** does not divide **b.**
Proof: by contraposition: Restatement of Theorem:
If a divides b evenly then a divides bc evenly where a,b,c $\in$ Z.

Since we have that $a|b$, by definition of divisible there exist an integer k so that $b = ak$.
Now consider $bc = (ak)c = a(kc) = am$ where $m = kc$ is an integer. Since $bc = am$, by definition of divisible, $a|bc$.
Since the contrapositive is true the equivalent original statement is also true and if **a** does not divide **bc** then **a** does not divide **b.**