Math 2534 Homework 3 Quantifiers Spring 2018

Follow the homework requirements **or points will be taken off**. In particular, stable multiple sheets.

Problem 1:

A) Put the following sentences in symbolic logic using a **single quantifier**.

a) Any child will play with trains
b) Not all students read the text book.
c) Everyone has a favorite movie.

Solutions:

a) Any child will play with trains

Let C be the set of all children (let x be an arbitrary child)
P(x) = x plays with trains.
$\forall x, x \in C \rightarrow P(x)$

b) Not all students read the text books.

Let S be the set of all students (let x be an arbitrary student)
P(x) = x reads the text book.
$\exists x | \; x \in S \wedge \sim P(x)$

c) Everyone has a favorite movie.

Let P be the set of all people (x be an arbitrary person)
M(x) = x has a favorite movie.
$\forall x, x \in P \rightarrow M(x)$

B) Negate b) in Part A symbolically and then convert to a natural English sentence.

$\sim [\exists x | \; x \in S \wedge \sim P(x)] \equiv \forall (x), \sim (x \in S) \vee \sim\sim P(x) \equiv \forall (x), \sim (x \in S) \vee P(x) \equiv \forall (x), x \in S \rightarrow P(x)$

All students read the text book.

Problem 2: Convert the logic statement into **natural conversational** English

a) Domain D: all professors
Predicate O(x) = x holds office hours.
$\forall x, x \in D \rightarrow O(x)$

All professors hold office hours

b) Domain B: all VT students
Predicate H(x) = x does homework..
$\exists x | x \in B \wedge \sim H(x)$

There is a VT student who does not do homework.

Problem 3: (You may put answer for this problem on this sheet)
Given the following domains and predicate, put each symbolic statement into natural conversational English.

Domain S: all student (x = an arbitrary student)
Domain M: all math classes (y = arbitrary math class)
Predicate: P(x,y) = x will register for y

1) $\forall y, y \in M \rightarrow [\exists x \big| x \in S \wedge P(x, y) \ ]$

Each math class has a student register for it.

2) $\exists x \big| x \in S \wedge [\forall y, y \in M \rightarrow P(x, y)]$

There is a student who registers for all math classes.

3) $\exists y \big| y \in M \wedge [\forall x, x \in S \rightarrow P(x, y)]$

There is a math class that all students must register for.

4) $\forall x, x \in S \rightarrow [\exists y \big| y \in M \rightarrow P(x, y)]$

Each student will register for at least one math class.

Problem 4: Put the following arguments into symbolic logic and determine their validity. (Justify your reasoning using sentences. State the **domain** and the **implied quantifier)**

Solution: Convert the sufficient condition to a positive statement:
"All good children get a treat."

A) No good child will miss out on a treat.
Billy is a good child.
Therefore Billy got a treat.

The domain G is the set of good children (x) with the implied Universal Quantifier.
The predicate is T(x) = x gets a treat. B represents Billy

$G(x) \rightarrow T(x)$

$G(B)$

$\therefore T(B)$

The argument is valid since Billy is in the domain.

B) No good child will miss out on a treat.
Amanda is not good child.
Therefore Amanda did not get a treat.

The domain G is the set of good children (x) with the implied Universal Quantifier.
The predicate is T(x) = x gets a treat. A represents Amanda

$G(x) \to T(x)$

$\sim G(A)$

$\therefore \sim T(A)$

The argument is not valid since Amanda is not in the domain.

C) No good child will miss out on a treat.
Liang did not get a treat.
Therefore Liang was not a good child.

The domain G is the set of good children (x) with the implied Universal Quantifier.
The predicate is T(x) = x gets a treat. L represents Liang.

$G(x) \to T(x)$

$\sim T(L)$

$\therefore \sim G(L)$

The argument is valid by the contrapositive.

D) No good child will miss out on a treat.
Dinesh got a treat
Therefore Dinesh is a good child.

The domain G is the set of good children (x) with the implied Universal Quantifier.
The predicate is T(x) = x gets a treat. D represents Dinesh.

$G(x) \to T(x)$

$T(D)$

$\therefore G(D)$

The argument is not valid by converse error.
There is no guarantee that Dinesh is in the domain.