Math 2534 Solution Homework 2 sec 2.1 – 2.3

Follow the Homework requirements. Put work on another sheet of paper.
Staple all multiple sheets.

Problem 1: Use Algebra of Logic to prove the following:

$[(p \to q) \wedge \sim p] \to [\sim (p \to q)] \equiv p$

Solution:

$Theorem: [(p \to q) \wedge \sim p] \to [\sim (p \to q)] \equiv p$ for propositons p,q.

Proof :

$[(p \to q) \wedge \sim p] \to [\sim (p \to q)] \equiv$	given
$\sim [(\sim p \vee q) \wedge \sim p] \vee [\sim (\sim p \vee q)] \equiv$	Implication law
$[(\sim\sim p \wedge \sim q) \vee \sim\sim p] \vee [\sim\sim p \wedge \sim q)] \equiv$	DeMorgan's law
$[(p \wedge \sim q) \vee p] \vee [p \wedge \sim q)] \equiv$	Double Negative law
$p \vee [p \wedge \sim q)] \equiv$	Absorption Law
p	Absorption Law

Therefore $[(p \to q) \wedge \sim p] \to [\sim (p \to q)] \equiv p$

Problem 2:

Using algebra of logic put the following in Disjunctive Normal Form

$(p \to q) \to [(p \wedge r) \to (q \wedge r)] \equiv$

Give a reason to validate each step below.

$(p \to q) \to [(p \wedge r) \to (q \wedge r)] \equiv$	Given
$\sim (\sim p \vee q) \vee [\sim (p \wedge r) \vee (q \wedge r)] \equiv$	Implication Law
$(p \wedge \sim q) \vee [(\sim p \vee \sim r) \vee (q \wedge r)] \equiv$	DeMorgan's Law
$(p \wedge \sim q) \vee (\sim p) \vee [(\sim r) \vee (q \wedge r)] \equiv$	Associative Law
$(p \wedge \sim q) \vee (\sim p) \vee [(\sim r \vee q) \wedge (\sim r \vee r)] \equiv$	Distributive Law
$(p \wedge \sim q) \vee (\sim p) \vee [(\sim r \vee q) \wedge T] \equiv$	Negation Law
$(p \wedge \sim q) \vee (\sim p) \vee (\sim r \vee q) \equiv$	Identity Law
$(p \wedge \sim q) \vee (\sim p \wedge \sim p) \vee (\sim r \wedge \sim r) \vee (q \wedge q)$	Idempotent Law

Problem 3: Put the following into symbolic **implication form**. Define all your variables.

a) I will clean up only if you help.
b) The game will be postponed if it is raining.
c) I will not go to the movie or I will not study.

Solution:

a) I will clean up only if you help.
Define C to be the statement " I will clean up"
Define H to the statement "you help".
$C \to H$

b) Game will be postponed if it is raining.
Define G to be the statement " Game is postponed"
Define R to the statement "It is raining".
$R \to G$

c) I will not go to the movie or I will not study.
Define M to be the statement " I will go to the movie"
Define S to the statement "I will study".
$\sim M \vee \sim S \equiv M \to \sim S$

Problem 4: If Anna goes to Roswell, New Mexico, she might see an alien space ship.

1) Rewrite the above sentence in converse form.
If Anna sees an alien space ship, then Anna went to Roswell.

2) Rewrite the above sentence in contrapositive form.
If Anna did not see an Alien ship, then she did not go to Roswell.

Problem 5: Determine if the following arguments are valid and justify your conclusion. Put each argument into symbolic logic and define all variables. In justifying your conclusion be sure to indicate what is the sufficient condition and what is the necessary condition.

a) If you are in the Marching Virginians , then you must go to the game.
You went to the game.
Therefore you are in the Marching Virginians.

Let M be the statement: You are in the Marching Virginians

Let G be the statement: you go to the Game.

$M \rightarrow G$

G

$\therefore M$

This argument is invalid. It is converse error since the necessary condition does not guarantee the sufficient.

b) If the test is Thursday, you will miss the game.
You did not miss the game.
Therefore you did not have a test.

Let T be the statement: Test is Thursday

Let G be the statement: you miss the Game.

$T \rightarrow G$

$\sim G$

$\therefore \sim T$

This argument is valid by the contrapositive.

Problem 6: P,Q and R represent the following statements:

P: Jim is a CS Major
Q Anne is an EE Major
R Laura is an Environmental Science Major
M Charlie is a Math Major

Assume that the expression $(P \vee \sim R) \rightarrow (Q \wedge M)$ is false and that R is true and M is true.

The implication, $(P \vee \sim R) \rightarrow (Q \wedge M)$, is false, so we know that the necessary condition, $(Q \wedge M)$, is false and the sufficient condition, $(P \vee \sim R)$, must be true. Since M is true, but $(Q \wedge M)$ is false means that Q must be false. We also know that R is true and therefore $\sim R$ Is false. The sufficient condition $(P \vee \sim R)$ is true so P must also be true for the disjunction to be tru. **So M is true, R is true, Q is false and P is true.**

Put the following statements into implication form and determine if the sufficient and necessary conditions are true or false and if the implication is true or false.

a) Anne is a EE Major or Charlie is not a Math Major.
$Q \vee \sim M \equiv F \vee \sim T \equiv F \vee F \equiv F$

b) Jim is a CS Major and Anne is not an EE Major.
$P \wedge \sim Q \equiv T \wedge \sim F \equiv T \wedge T \equiv T$

c) Only if Anne is a EE Major is Jim a CS Major
$P \rightarrow Q \equiv T \rightarrow F \equiv F$

Problem 7: **A Logic Puzzle and Working with Hypothesis**

For your solution to the logic puzzle below, you need to put all statements into symbolic logic and define your variables. Then write a concise **paragraph** that **justifies** your reasoning and your conclusion. Indicate clearly the sufficient and necessary conditions. **Summarize your conclusions.**

Report Card

Three siblings Alice, Bob and Carol truthfully reported their grades to their parents as follows:

Alice: If I passed math, then so did Bob.
I passed English if and only if Carol did.

Bob: I passed math only if Alice did.
Alice did not pass History.

Carol: Either Alice passed history or I did not pass it.
If Bob did not pass English, then neither did Alice.

If each of the three passed at least one subject and each subject was passed by at least one of the three, and if Carol did not pass the same number of subjects as either of her siblings, which subjects did they each pass?

Define your variables as follows so that we all have the same notation:

A_E means that Alice passed English

A_M means that Alice passed Math

A_H means that Alice passed History

B_E means that Bob passed English

B_M means that Bob passed Math

B_H means that Bob passed History

C_E means that Carol passed English

C_M means that Carol passed Math

C_H means that Carol passed History

Solution:

Alice passed Math and English. Bob passed all three subjects. Carol passed English.

Bob states that Alice did not pass history. Carol states "Either Alice passed history or I did not pass it." Since we already know that Alice did not pass history, then the statement that She did pass history is false. So the Exclusive Or indicates that Carol's statement "I did not pass it must be true. So Carol did not pass history. Since every subject was passed by at least one person, Bob passed history.

Now consider Carol's statement " If Bob did not pass English, then neither did Alice." And Alice's statement "I passed English if and only if Carol did. " If it were true that Bob did not pass English, then this sufficient condition guarantees that Alice died not pass English as well. If Alice did not pass then by the "if and only if" statement we know that Carol also did not pass. (since the two statements must have the same truth value of false.)
However this means that no body passed English. But very subject was passed by at least one person, so this means that everyone passed English.

Alice states that "If I passed math, then so did Bob." And Bob states that "I passed math only if Alice did.". This is equivalent to stating that " Alice passes Math if and only if Bob passes Math.". Suppose that Alice and Bob do not pass math. Then Carol must have passed math since every course is passed by at least one person. This means that Carol will pass two classes, Alice passes one and Bob passes two. But Carol does not pass the same number of subjects as either of her siblings. Therefore Alice and Bob did pass math and Carol did not.