Math 2534_ In Class_ PMI_ worksheet

Problem 1:

Theorem: $if\ f(x) = x^2 e^x$, then $f^{(n)}(x) = e^x[x^2 + 2nx + n(n-1)], \forall n \in N, x \in R$

Problem 2:

Theorem: $n! > 4^n$ for natural numbers n >? (fill missing value for n before beginning proof)

Problem 3: Theorem: $\forall n \varepsilon N$, $3^n - 1$ is divisible by 2

Problem 4:

Theorem: For all natural numbers $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}^n = \begin{bmatrix} 2^n & n2^{n-1} \\ 0 & 2^n \end{bmatrix}$

Problem 5:

Theorem: The function $f(x) = 2x^2 + 6x + 5$ is even for all values of the natural numbers.

Proof by PMI:

If my hypothesis is true for n = 1, then assume true up to some arbitrary k so that $f(k) = 2k^2 + 6k + 5$ is even and then prove true for k + 1: i.e. we will show that $f(k+1) = 2(k+1)^2 + 6(k+1) + 5$ is also even. Consider the k + 1 term given below:

$f(k+1) = 2(k+1)^2 + 6(k+1) + 5 = 2k^2 + 4k + 2 + 6k + 6 + 5 = (2k^2 + 6k + 5) + 4k + 6$

$= (2k^2 + 6k + 5) + 2(2k + 3)$

We assumed $2k^2 + 6k + 5$ is even and 2(2k + 3) = 2m is even by definition of even where m = 2k+3 is an integer. Therefore $(2k^2 + 6k + 5) + 2(2k + 3)$ is even since the sum of two even integers is always even.

I have assumed true for k and proved true for k+1, therefore $f(x) = 2x^2 + 6x + 5$ is even for all natural numbers.