Math 2534 Homework 12 Functions and Equivalence Relations

Show all work and staple multiple sheets.

Problem 1:
Given $A = \{a, b, c, d\}$, $F : X \to Y$ for X is the set $P(A)$] and Y =$\{0,1,2,3,4,5,6\}$,
Define F on all elements S in P(A) so that F(S) = n(S) (ie. the number of elements in S)

a) Is F one to one? Justify your conclusion
b) Is F onto? Justify your conclusion

Problem 2 : Explain the mistake in the following proof:
Theorem: If $f(x) = 5x^2 - 2$ for all integers, Then *f(x)* is one to one.
Proof: Suppose any integer z is given. Then by definition of a function, there is only one possible value for *f(z)*, namely y = $f(\text{z}) = 5\text{z}^2 - 2$. Hence *f (x)* is one to one.

Problem 3: For each of the following relations defined on the set A = { 1,2,3,4,5}, determine if R is reflexive, symmetric and/or transitive. Draw the directed graph for each relation R . If R is an equivalence relation the give the partition of A.

$R1 = \{(1,1),(2,2),(2,3)(3,2),(3,3),(3,4),(4,3),(5,5),(4,4)\}$
$R2 = \{(1,2),(1,4),(1,5)(2,4),(2,5),(3,4),(3,5),(4,5),(4,4)\}$
$R3 = \{(1,3),(1,5),(2,4)(3,1),(3,5),(4,2),(5,1),(5,3)\}$
$R4 = \{(1,1),(2,2),(1,3),(1,5),(3,1),(3,3),(2,4),(5,3),(3,5),(4,2)(4,4),(5,1)\}$

Problem 4: Prove or give a counter example;
Theorem:
If the relations S and R are each transitive then the union $\text{S} \cup \text{R}$ is also transitive.

Problem 5: Prove the following:
Theorem: A relation R is a symmetric relation on the set of all sets when for sets A,B, ARB if and only if there is a bijection from one set to other.

Problem 6: Modular Equivalences: Let a, b be any integers and d is a positive integer where d > 1. The following statements are equivalent.

$a)$ $a \equiv b \bmod d$

$b)$ $a \bmod d = b \bmod d$

$c)$ $d \mid a - b$

$d)$ b = dq + $a[\bmod d]$, for $q \in Z$

Part A: Prove the following:

Theorem: If R is defined on the integers so that $a \equiv b \bmod 6$, then R is an equivalence relation on the integers when for integers a, b, aRb iff $6 \mid a - b$.

Part B:

Zmod 6 partitions the integers as follows: $Z = [0] \cup [1] \cup [2] \cup [3] \cup [4] \cup [5]$

Label the following equivalence classes with the correct remainder representation.

$[8], [-9], [24], [-22]$