Math 2534 Homework 11 Spring 2018

Show all work and staple multiple sheets.

Problem 1: Given elements a, b in the Boolean algebra B with operations ⊠ and ⊙ where m is the identity for ⊠ and p is the identity for ⊙, Justify each step of the proof below . The inverse of any element **a** is **a**$'$.

Theorem: For a, b in B, (a ⊠ b)$'$ ⊙ (b ⊠ b) = m

Problem 2: Consider the set A of all divisors of 18, A = { 1, 2, 3, 6, 9, 18}, with the operations defined as follows: a ♥ b = LCM(a, b), a ♦ b = GCD(a,b) and the complement(or negation) is defined to be $a^C = 18/a$.

1) Find the elements in this set that will act as the identity for each operation.
2) Determine if the universal bound property holds for these operations
3) Determine if the complement (inverse) property holds for these operations.
4) Intuitively, do you think this system forms a Boolean algebra? Why or Why not?

Problem 3: Prove the following using method of contradiction:

Theorem: Let A and B be finite sets and n(A) < n(B). If f maps A to B, then f cannot be onto.

Problem 4: Let the function h(x) map set A to set B. Let C and D be disjoint subsets so that $C \cup D = A$. Define functions $f: C \to B$ and $g: D \to B$ so that h(x) = f(x) for all x in C and h(x) = g(x) for all x in D.

Determine if the following are true statements. Justify your conclusions.

a) If f(x) is one to one and g(x) is one to one, then h(x) one to one.
b) If $f^{-1}(x)$ exist and $g^{-1}(x)$ exists, then $h^{-1}(x)$ exists.

Problem 5: If functions f and g are each onto then the composition is also onto.

Problem 6: If the function f(x) maps A to B and $f^{-1}(x)$ exists, then f(x) is onto.

Problem 7:

If $f: A \to B,\ g: B \to C$ and $(f^{-1} \circ g^{-1})(4) = f^{-1}(2)$ and $g(b) = 4,$ then find the value of b.