Math 2534 Homework 10 Spring 2018

Show all work and staple multiple sheets.

Problem 1:

Use Proof By Set Algebra to prove the following where A, B are Sets
Justify each step. Convert to union and intersection operations only.

$[A^C \cup (B - A)]^C - A^C = A$

Problem 2: Define a Boolean Algebra as follows:
Let B = { 1, 2, 3, 5, 6, 10, 15, 30} be the set of all positive divisors of 30 with operations defined on the set to be as follows: a + b = LCM(a,b) , and the $a \bullet b$ = GCD(a, b)

The complement is defined $\bar{a} = \frac{30}{a}$

a) Determine the identities for each operation.
b) Evaluate $5 + \bar{5}$ and $3 \bullet \bar{3}$ and determine if the results are correct for a Boolean algebra.
c) Determine if DeMorgan's Law is valid for the following: $\overline{(6+15)} = \bar{6} \bullet \overline{15}$

Problem 3: Fill in the reasons for each step for the following.

Theorem:

For all a in the Boolean Algebra B under the operations of $+, \bullet$, $(a + \bar{a}) + \overline{(h \bullet a)} = k$, where k is the identity for operation $\bullet$ and h is the identity for +.

Proof:

$(a + \bar{a}) + \overline{(h \bullet a)} =$	____Given________
$k + \overline{(h \bullet a)} =$	___________________
$k + (\bar{h} + \bar{a})$	___________________
$k + (k + \bar{a}) =$	___________________
$k + k =$	___________________
k	___________________

Problem 4:

Let a and b be elements in the Boolean Algebra B with the following defined operations. Operation 1 is $\otimes$ with identity $\boldsymbol{p}$ and operation 2 is $\odot$ with identity $\boldsymbol{q}$. The complement (or negative) is represented by $\bar{a}$. Prove the following:

$$\bar{a} \otimes [\overline{(a \otimes b)} \otimes (b \odot \bar{q})] = \bar{a}$$