

Practice session 11/30/2018

1. (Inspired by Putnam 1968, B6) Prove that a polynomial with only real roots and all coefficients equal to ± 1 has degree at most 3.

2. (Putnam 1974) Call a set of positive integers “conspirational” if no three of them are pairwise relatively prime. What is the largest number of elements in any conspirational subset of integers 1 through 16?

3. (From Putnam 1942, problem A-3) Is the following series convergent or divergent?

$$1 + \frac{1}{2} \cdot \frac{19}{7} + \frac{2!}{3^2} \left(\frac{19}{7}\right)^2 + \frac{3!}{4^3} \left(\frac{19}{7}\right)^3 + \frac{4!}{5^4} \left(\frac{19}{7}\right)^4 + \dots$$

4. Prove that

$$|\sin(nx)| \leq n|\sin x|$$

for any real number x and positive integer n .