Practice session 11/14/2018

1. Prove that $n! < \left(\dfrac{n+1}{2}\right)^n$, for $n = 2, 3....$ Hint: Use the Arithmetic Mean-Geometric Mean inequality.

2. A spherical, $3-$dimensional planet has center at $(0,0)$ and radius 20. At any point of the surface of this planet, the temperature is $T(x, y, z) = (x + y)^2 + (y - z)^2$ degrees. What is the average temperature of the surface of this planet?

3. Show that the equation
$$n_1^4 + n_2^4 + \cdots + n_{14}^4 = 1599$$
has no solutions in nonnegative integers. Hint: Think mod 16.