Practice session 11/2/2018

1. (Leningrad Mathematical Olympiad, 1988) Let $f : \mathbb{R} \to \mathbb{R}$ be continuous, with $f(x) \cdot f(f(x)) = 1$ for all $x \in \mathbb{R}$. If $f(1000) = 999$, find $f(500)$.

2. If $a, b, c \geq 0$, prove that $\sqrt{3(a+b+c)} \geq \sqrt{a} + \sqrt{b} + \sqrt{c}$.

3. If p is an odd number not divisible by 3, then it is congruent to ± 1 modulo 6.

4. Suppose $p > 0$ is prime. Show that $p^2 + 2$ cannot be prime. Hint: Use #3.

5. Define a *domino* to be a 1×2 rectangle. In how many ways can an $n \times 2$ rectangle be tiled by dominoes?