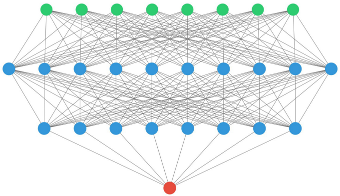
Deep Learning for Parameter Estimation in Neuronal Dynamics

Goal: Computationally learn an inverse map that estimates parameters in physical and statistical models

- Neuronal dynamics (ODE):
  $$\frac{du}{dt} = \gamma\left(u - \frac{u^3}{3} + v + \zeta\right), \quad \frac{dv}{dt} = -\frac{1}{\gamma}\left(u - \theta_0 + \theta_1 v\right)$$
- Correlated noise (AR):
  $$\eta(t_i) := \rho\, \eta(t_{i-1}) + \epsilon(t_i), \quad \eta(t) \sim \mathcal{N}\left(0, \sigma^2/\Delta_t^2\right)$$
- Observational data $d(t) = u(t) + \eta(t)$, parameters $m = (\theta_0, \theta_1, \sigma, \rho)$

Illustration of (artificial) neural network realizing the inverse map:

$\tilde{\mathcal{F}}^{-1}_{\text{training data}} : d \mapsto m$

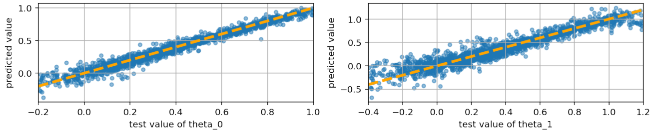


(Image source: wikimedia.org)

Challenges: Highly nonlinear dynamics and inference characteristics; Orders-of-magnitude difference in time scales in physical model (ODE) vs. statistical process (AR)

Significance: Joint inference of parameters of (deterministic) physical models and of statistical models is rarely attempted with traditional methods; but estimation of noise can be relevant in cases where noise is unknown a-priori

Accuracy of estimation of parameters θ_0, θ_1 (from testing data set).



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References I

Rudi, Johann, Julie Bessac, and Amanda Lenzi (2022). "Parameter Estimation with Dense and Convolutional Neural Networks Applied to the FitzHugh–Nagumo ODE." In: *Proceedings of the 2nd Mathematical and Scientific Machine Learning Conference*. Ed. by Joan Bruna, Jan Hesthaven, and Lenka Zdeborova. Vol. 145. Proceedings of Machine Learning Research. PMLR, pp. 781–808.