Frank Quinn overheads Jan 2005

History of manifolds

Long complex development, huge literature, hopefully illuminates:

- what are mathematical objects, and
- how do we come to understand them?

Objective is to improve mathematical practice.

1845 The term introduced (Grassman, Plücker)

higher dimensional linear space

1855 Riemann

- Philosophical, nonlinear, setting for Riemann surfaces
- organizing concept, "useful language" (Poincaré)

1900 Poincaré

- defined by example
- unsubstantiated claims of remarkable properties
- solid theory only in dimension 2.
- strange mix of techniques
- consolidation inhibited by desire for coherent global picture
- van der Waerden "battlefield of methods"

1930 definitions of smooth, homology manifolds

• gave up on immediately seeing unified picture, though developmer of PL probably inhibited by expectation there would **be** a unified picture.
• definitions justified by technical success
• **descriptive** theory of **individual** objects

1955 Thom, Smale, Milnor

• now expected H*, top, pl , smooth all to be different; different families have very different techniques.
• precise relations top/pl/smooth worked out,
• focus shift to behavior of w**hole families** of manifolds
• **constructive** techniques

1970 "Expert world view" Sullivan, Wall

• divide into high/low dimensions rather than H*/top/pl/smooth
• high (≥ 5) dimensions have elaborate "characteristic behavior" witl class a perturbation on this
• low dimensions are ≤ 3 (reasonable theory) and 4 (mystery)
• community focus shifts to low dimensions

1980 view sharpened

• Freedman: topological 4-manifolds largely exhibit "high dimensional" behavior
• Donaldson: smooth 4-manifolds really strange (weak descriptive theory)

1995 view sharpened

• Homology manifolds exhibit high dimensional manifold behavior (Q, BFMW)
• Seiberg-Witten invariants of smooth 4-manifolds

2004 view sharpened

• Perelman geometrization
• Oszvath-Szabo Floer theory
• "big" topological 4-manifolds?

Next:

• structure theory for smooth 4-manifolds? (Poincaré conjecture, classification)
• are there two **low** dimensions (≤ 3 and smooth 4) or is there a single characteristic low-dimensional "behavior" as in high dimensions?
• new theory of smooth high-dimensional manifolds?
H*/top/pl behavior qualitatively simpler than smooth; see past this?