MATH / MATH-ED TERMINOLOGY PROBLEMS

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ABSTRACT. Many common terms have very different meanings in the two communities, and sometimes neither is appropriate. Actual solutions will require us to recognize and transcend terminology problems.

1. A Search for Meaning

A few years ago a draft K–12 Standards Document arrived at the AMS for review. This happens from time to time and while as far as I can tell AMS feedback has no effect, it is flattering to be asked. However this Document was accompanied by a guide for reviewers that included the question:

"Do the standards specify a range of cognitive skills to be expected, including some range of the following?

- Remembering: recognizing, recalling
- Understanding: selecting, interpreting, illustrating, classifying, summarizing, inferring, comparing, explaining
- Applying: using, executing, implementing, computing, translating
- Analyzing: differentiating, organizing, attributing, synthesizing
- Evaluating: checking, critiquing, justifying
- Creating: generating, hypothesizing, planning, designing, constructing"

Say what?? Are these ranges of cognitive skills or ranges of synonyms?

Math educators generally reject use of careful definitions so one cannot just look these up. However there is an extensive literature from which we could try to infer meanings, and we can see how these things actually play out in students. Two conclusions emerge: first, as expected, these are for the most part synonyms and reflect a richness of language rather than of content. A more troubling conclusion is that when when these terms do have specific meanings they are quite different from the meanings used in the mathematical community[1].

2. Misunderstanding Understanding

Every discipline develops terminology adapted to the discipline. Specialized meanings for common terms lead to "talking past each other" communication failures. We illustrate this with the term "understand".

The mathematical community has evolved a rather strong meaning for "understand": roughly "complete mastery" including full facility with working problems. Weaker meanings have been found to be dysfunctional in the sense that they do not provide a foundation for further mathematical learning.

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[1]For a more detailed discussion with a different objective see Communication between the mathematical and math–education communities.

The educational community has a much weaker meaning for this term. My guess is that it reflects something about human learning: people learn some things (e.g. inferring patterns from examples) quickly and easily. Fixing errors in this natural learning is a different process and much harder, so it makes sense to have terms for the first step. “Understand” may be one of these. At any rate the math–ed meaning for “understand” is closer to “show evidence of exposure”. Teachers can say “you can’t work the problems but I see that you basically understand, so I can give you partial credit”. And when students get to the college level they say “I really do understand it, but just can’t work problems. Can’t you give me partial credit?”

There are similar mismatches with most other terms. Does “recall the quadratic formula” mean “know and be able to use the quadratic formula” or “recall having seen the quadratic formula”? Does “know multiplication facts” mean “know there is a multiplication table” or “be able to multiply numbers with facility”? Terms such as “synthesizing”, “justifying”, “creating”, “discovering”, etc. refer to highly-structured activities that have little in common with the mathematical meanings.

3. Right, Wrong or Different?

To a degree these terminology issues can be seen as cultural: they have their meanings, we have ours, and it is neither necessary nor appropriate to declare one or the other “wrong”. We just have to be mindful of the differences and very careful when trying to communicate.

There are, however, cases where one meaning really is wrong. The slogan “we should put less emphasis on rote learning and mechanical calculation, and more emphasis on understanding” has strongly influenced math education in the last few decades. It is certainly very attractive. But remember that there is a job to be done: students should emerge with a good foundation for further mathematical learning. We are not at liberty to use any convenient meaning for “understand” but must use one that actually gets this job done. The math–ed meaning is dysfunctional in this regard and so—in the context of the slogan—is actually wrong.

I do not believe that use of a dysfunctional meaning for “understand” is an evil plot designed to cripple higher education in mathematics, even if it is working out that way. The K–12 system is rather self–contained and the curriculum adapts to whatever students can do. The failure to “provide a foundation” only becomes unavoidable and acute at the college level. K–12 educators are pretty unresponsive to complaints from the college level, but in their defense it must be said that these complaints are often incoherent.

4. Plea

It is important to realize that the real problems will not have terminology solutions. Mathematical understanding is too demanding to be appropriate at the school level. The mathematically–adapted meaning for “understand” might make the “understanding, not rote calculation” slogan correct but would also make it unrealistic. The proper goal may be a mostly-subconscious *template* for mathematical understanding. The math–ed meaning will likely play an important role in it’s development. Making effective sense of something like this would take deep insights into both human learning in general and the needs of long-term learning in mathematics. It is likely to require cooperative effort by both the math and math–ed communities.

Unfortunately, we won't be able to formulate or agree on the real problems, much less solve them, until we sort out the terminology issues.