CMDA 4604: INTERMEDIATE TOPICS IN MATHEMATICAL MODELING

Tentative Schedule

The pace of the course and topics covered will be adjusted as the semester progresses, but the schedule below gives a rough impression of the territory we seek to cover.

				topic	*take-away*
August	24	M	1.	overview; derive heat eqn	PDEs model physical phenomena
	26	W	2.	heat equation; steady-state heat	PDEs model dynamic and static conditions
	28	F	3.	linear operators, function spaces	Linear PDEs resemble linear algebra's $Ax = b$
	31	M	4.	inner products, projectors	'geometry' for function spaces
September	2	W	5.	best approximation theorem	compute optimal approximations from subspaces
	4	F	6.	spectral method for matrices	solve $Ax = b$ using eigenvalues, vectors
	7	M		*Labor Day*	
	9	W	7.	finite difference discretization	diff eqs can be approximated by matrix equations
	11	F	8.	eigenvalues of the Laplacian	operators have eigenvalues, eigenfunctions too
	14	M	9.	spectral method for Laplace's eqn	solve $u'' = f$ using eigenvalues, functions
	16	W	10.	spectral method, continued	solve $u'' = f$ using eigenvalues, functions
	18	F	11.	boundary, initial conditions	boundary conditions affect eigenvalues, functions
	21	M	12.	weak form of Laplace's eqn	impose the weak form on a finite dimensional space
	23	W	13.	Galerkin method	PDEs have 'variational' formulations
	25	F	14.	essential/natural boundary conds	some boundary conditions are 'natural' to the weak form
	28	M	15.	Galerkin with hat functions	the classic 'finite element method'
	30	W	16.	stiffness matrix properties	matrix eigs approximate Laplace eigs
October	2	F	17.	interpolation	approximate functions with piecewise polynomials
	5	M	18.	quadrature rules	approximate integrals for FEM matrices, vectors
	7	W	19.	finite element error analysis	convergence rate depends on finite element space
	9	F	20.	spectral method for heat eqn	time dependence reduces to independent ODEs
	12	M	21.	boundary conditions for heat eqn	boundary conditions add minor complications
	14	W	22.	recap; review for midterm	
	16	F		*Fall break*	
	19	M	23.	FEM for heat eqn, exact in time	Galerkin, time dependence gives $u' = Au + f$
	21	W	24.	matrix exponential, forward Euler	e^{tA} is exact but expensive
	23	F	25.	forward Euler for heat eqn	forward Euler is fast but unstable
	26	M	26.	stability, backward Euler	backward Euler is stable, but requires linear solve
	28	W	27.	eigenvalues of Euler matrices	eigenvalues reveal everything about Euler behavior
	30	F	28.	spectral method for wave eqn	wave equation gives $u'' = Au + f$
November	2	M	29.	first order form of wave eqn	imaginary eigenvalues; conservation of energy
	4	W	30.	FEM for wave eqn	Which methods conserve energy of the solution?
	6	F	31.	nonlinear wave eqns	solutions are constant on characteristic curves
	9	M	32.	nonlinear wave eqns	nonlinear waves can form shocks
	11	W	33.	time stepping for nonlinear waves	implicit methods give a nonlinear system
	13	F	34.	Newton's method for systems	numerical solution of implicit problems
	16	M	35.	nonlinear optimization	overview of basic methods
	18	W	36.	nonlinear optimization	application: finding steady-state solutions
	20	F	37.	nonlinear optimization	application: linear stability analysis
	23	M		*Thanksgiving break*	
	25	W		*Thanksgiving break*	
	27	F		*Thanksgiving break*	
	30	M	38.	UQ = uncertainty quantification	How does uncertain propagate in a simulation?
December	2	W	39.	UQ: frequentist approach	model uncertainty in a crude manner
	4	F	40.	UQ: Bayesian approach	model uncertainty via Galerkin for Laplacian
	7	M	41.	UQ: pendulum/circuit example	put these models of uncertainty to the test
	9	W	42.	recap; review for final	