“Useful Is As Useful Does”

Ezra Brown, Alumni Distinguished Professor of Mathematics
Virginia Tech, Blacksburg VA 24061
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“Does it have an application?” “What’s it good for?” “Does this stuff have any use?” Every teacher of mathematics hears these questions over and over from students, especially students at a technical institution, who really want to know the answers.

To the leaders of ancient Greece, this was a no-brainer: mathematics was clearly a useful discipline. The word they used was ***mathemata***, which meant *that which is to be learned.* Mathematics – the *quadrivium* of arithmetic, geometry, astronomy and music, and the *trivium* of grammar, rhetoric and logic – trained minds to think critically and analytically, to examine assumptions and to argue from premise to conclusion. These were the tools of statecraft, of diplomacy, and of law. If you would ask a citizen of Athens, “Is mathematics useful?” the reply would invariably be, “Of course. Isn’t that obvious?”

But “useful” doesn’t exactly mean that nowadays, does it? No. For us twenty-first century folks, utility lies in the application of a subject to the making of things, applications to engineering, computer science, economics, chemistry, physics, biology and the like. We want our math teachers to tell us something that we can use to design better buildings…to build better computers…to create faster networks and neater cell phones and better pharmaceuticals…to make money.

Consider, then, the plight of mathematicians who revel in the beauty and the elegance and the music of mathematics. (And yes, mathematics is musical: It has rhythm, melodies, themes and variations, color, depth, brilliance, and harmony that everyone can discover. But that’s another story.) How do mathematicians justify what we do in this increasingly utilitarian world, especially those of us who specialize in such arcane fields as algebraic geometry and number theory?

My answer is this: sometimes it takes a while for the application to appear. Then I give the questioner some examples. (Mathematicians love examples). Here are three:

The Greek mathematician Menaechmus, in around 350 BCE, was the first to describe the ellipse, the parabola and the hyperbola. He did not see any immediate use for these conic sections, except as an aid to making certain geometrical construction.

Two centuries later, Apollonius of Perga wrote a nine-volume treatise called *On Conics*, and he was criticized for studying things that were clearly useless. He had an answer: he said he didn't care, and that they were worthy of study in their own right because they were beautiful and pleasant.

But they were useful, and 1800 years later, they found their uses in the astronomy of Kepler, in Galileo's Law of Falling Bodies, in Newton's mechanics, in Newton's telescope that used the reflecting properties of parabolas, and in many other places. Finally, in the late twentieth century, many sufferers of kidney stones have had these stones pulverized by a noninvasive procedure called *lithotripsy*, which uses the reflective properties of ellipses.

G. W. Leibniz, one of the inventors of the calculus, came up with a clever way of writing numbers in 1679. He devised a positional notation just like the one we use today, except that it was base two, not base ten. That is, instead of writing one hundred and seventeen as 1*100 + 1*10 + 7*1, it is written as sums of powers of two, namely

117 (base ten) = 64 + 32 + 16 + 4 + 1 = 1110101 (base two).

Nobody could see any possible use for this scheme, and it remained a curiosity, until a few inventors in the 1930s saw how information could be encoded as binary numbers – and this gave birth to the digital computer and all our zillions of modern electronic marvels.

Finally, one of the most useless but beautiful areas of mathematics was number theory, the study of the beauty and the splendor and the wonder and the magic of the numbers: 1, 2, 3, … The Euler-Fermat Theorem epitomized this useless beauty. It is an elegant result (never mind what it is, exactly), due to the seventeenth-century French mathematician Pierre Fermat and generalized by the eighteenth-century Swiss mathematician Leonhard Euler. It remained totally useless until the 1970's when, much to everyone's surprise, it became a powerful tool that led to the establishment of one of the most active areas of applied mathematics on the planet. I'm referring to public key cryptography, the art and science of safeguarding information and of computer and network security!

We may not immediately see the use of a mathematical result or theory, and it might take a while for an application to show up, but don't dismiss it. Useful is as useful does!