Richard Guy, Bud Brown, and the Making of *The Unity of Combinatorics*

Ezra Brown

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1 Introduction

This is a story about how the eminent mathematician Richard Guy wrote a paper [5] in 1995 about the numerous connections between and within the many areas of combinatorics, and how the paper was turned into a book [3] published in 2020, with the help of Bud Brown, a mathematician and a mathematical expositor. We'll learn about Richard and why he wrote the 1995 paper and how Don Albers, a book acquisitions editor, wanted Richard to expand the paper into a full-length book. We'll learn about Bud and why Don encouraged Richard to seek the help of a coauthor — specifically, Bud. Along the way we'll see some of the mathematics, complete with connections. Finally, we'll learn something about what's involved in transforming a thirty-page paper into a 350-page book.

So, let's talk about Richard.

2 Richard Guy, mathematician extraordinaire

Richard Kenneth Guy was born September 30, 1916 in Warwickshire, England and was the only child of two schoolteachers. After education at Cambridge (Caius College) and Birmingham University he began teaching at a British grammar school. Eventually, his mathematical interests and talents led him to a university career that took him from London University to the University of Malaya, Singapore, to the Indian Institute of Technology in New Delhi and, finally, in 1965, to the University of Calgary where he spent the rest of his life.

Richard's mathematical interests were broad. He wrote some 300 papers on number theory, geometry, combinatorics, graph theory, recreational mathematics, and combinatorial game theory – of which he was one of the co-founders. He mentored hundreds of students and colleagues, and showed great interest in all kinds of mathematics. His place at mathematical talks, seminars, and colloquia was invariably in the front row of the audience.

We will perhaps best remember him through his dozen or so books. His fifteen-year collaboration with Elwyn Berlekamp and John Conway resulted in the four-volume work

Winning Ways for Your Mathematical Plays, which introduced the mathematics community to the wonders of combinatorial games and Sprague-Grundy theory (which Richard had independently rediscovered). We wlll look at some of those combinatorial games later.

He was an inveterate collector of interesting problems and he had a terrific ability to frame them as intriguing, appealing mysteries; his *Unsolved Problems in Number Theory* is a beguiling invitation to that discipline. He possessed a longstanding interest in chess endgames; for a time he edited a column in a British chess magazine and he maintained an encyclopedic archive of problems.

Richard had a sense of humor composed in equal parts of wordplay, mathematical cleverness, and self-deprecation; to read any of his books or articles is to experience that sense of humor. In person he would flash a mischievously proud grin when he pulled one of these off. In his award-winning article, "The Strong Law of Small Numbers," [6] he exhibited thirty-five examples; some were true patterns and the rest were coincidences. He wittily explained to us that there just are not enough small numbers to handle all the demands we place upon them! We will look at a few of these examples just to give you the flavor of this mathematical gem.

Of late he had turned his attention to triangle geometry and to finishing his Carus monograph *The Unity of Combinatorics* – or *TUoC*, for short – which was published in May 2020. *TUoC* began in 1995 as a 30-page paper [5] of the same name. The latter was an outline of a proposed lecture series, whose purpose was to feature the many connections within the vast area of combinatorics, thereby dispelling the then-prevalent notion that combinatorics is just a bag of tricks.

So, let's talk about the book, and then we'll look at examples of these connections from the beginning of the *TUoC* paper.

3 Inside the *TUoC* book

The *TUoC* book [3] contains topics from several broad areas of combinatorics; these include some familiar items and some that are not so well known, as follows:

- Sequences: Fibonacci numbers, Catalan numbers, Mersenne numbers, Pascal's triangle, Langford, Skolem, Beatty, and de Bruijn sequences, Penrose tilings, Conway worms, Chebyshev polynomials, sequences from Newton's method
- Combinatorial games: nim, Wythoff's game, coin-turning games, product games, Turning Turtles, Mock Turtles, Moebius, Mogul, nim addition, nim multiplication
- Combinatorial designs: Latin squares, orthogonal Latin squares, magic squares, block designs, difference sets, Kirkman's schoogirls problem, Singer designs, Steiner systems, automorphism groups of combinatorial designs, the Miracle Octad Generator
- Graph theory: Euler circuits, graph colorings, complete graphs, the Four-Color Theorem, Heawood's conjecture, the Heawood graph, perfect squared squares, Euler's polyhedral formula for planar graphs, dual graphs, packings and coverings of graphs

- Finite geometries: finite projective geometries, finite affine geometries
- Error-correcting codes and related topics: Hamming codes, Golay codes, perfect codes, incidence matrices, Hadamard matrices, sphere-packing, the Kepler Conjecture
- Algorithms for finding such optimal structures in graphs and networks as minimal connector trees, minimal spanning tree, maximal flows in capacitated networks, Euler circuits, maximal matchings in bipartite graphs, and other greedy algorithms, all of these being associated with matroids
- Number-theoretic connections: sums of 1, 2, 4, and 8 squares, quaternions, octonions, the non-negative integers as a field

Richard conceived of the book as a number of walks through the Combinatorial Garden, showing connections on every hand. There were about sixty topics in all, and all but a dozen of those appear in the book. Other material was added in support of one or more of the original topics. Topics were moved around from the original order as seemed appropriate.

About half of the book (eleven chapters) consists of previously published papers. Eight of these are by either Richard or Bud, some with coauthors and some without, and three other papers are by outside authors. These papers have been suitably modified to fit the style and the flow of the book, and all of them were liberally sprinkled with cross-references and sign posts, such as “Look in Section 7.1 for an introduction to difference sets” or “We defined these objects back in Chapter such-and-such, but it doesn’t hurt to review them here.” Some diagrams and figures are repeated because they arise in many different contexts. These chapters added about twenty more topics to the book.

The remaining eight chapters are Guy-Brown collaborations, each of which began as short sections in *TUoC* – the paper. To give two examples, the first six pages of the paper grew into the forty-two pages of the book’s Chapters 1, 2, and 3, and the two pages on difference sets in the paper became the thirteen pages of Chapter 7 in the book.

With that, let’s go to the beginning of the *TUoC* book and describe some of its combinatorial connections.

4 The *TUoC* book and some of its combinatorial connections

A small child playing with colored blocks on the floor has an interesting idea. The youngster arranges six blocks in a row, placing one block between two reds, two blocks between two greens, and three blocks between two blues like this: **B R G R B G**. Let’s use numbers, so that the sequence looks like this: **3 1 2 1 3 2**. Can you do this for four pairs? Yes, it’s not hard to find **4 1 3 1 2 4 3 2**.

Can you do this with one or two pairs? No. How about five pairs, or six pairs, or n pairs in general? It turns out that if such a sequence is possible for n pairs, then

$n \equiv -1$ or $0 \mod 4$, and there are algorithms to generate such sequences for any number of pairs $n = 4k - 1$ or $4k$.

We call these arrangements of pairs of numbers *Langford sequences*, named for the child's father Dudley Langford, who watched as the kid arranged the original six blocks. Langford published this account as a puzzle in the journal *Mathematical Gazette* in 1958, it generated considerable interest, and Langford sequences and their generalizations are currently thriving areas of research.

Among the interested parties was Thoralf Skolem, who solved a problem he had posed by modified the Langford sequences as follows: Beginning with **3 1 2 1 3 2**, he appended a pair of zeros to get **3 1 2 1 3 2 0 0**, then added 1 to each number, obtaining **4 2 3 2 4 2 1 1**. What results is a *Skolem sequence*, i.e. a arrangement of $2n$ pairs (k, k) in which every pair of k's is separated by $k - 1$ intervening numbers.

Next, he made a two-row array by writing the numbers 1 through 8 under his Skolem sequence, obtaining

4	**2**	**3**	**2**	**4**	**3**	**1**	**1**
1	2	3	4	5	6	7	8

.

and observed that on the bottom line, 1 and 5 are 4 places apart, 2 and 4 are 2 apart, 3 and 6 are 3 apart, and 7 and 8 are 1 apart. This yields the four pairs $(5, 1), (4, 2), (6, 3),$ and $(8, 7)$, as well as the equalities

$$8 - 7 = 1, 4 - 2 = 2, 6 - 3 = 3, \text{ and } 5 - 1 = 4, (*)$$

thus solving that problem Skolem was working on, namely to partition the numbers from 1 to $2n$ into pairs whose differences are the numbers 1 through n. You give Skolem a Langford sequence on $n - 1$ pairs and his construction will produce the required partition of the numbers 1 through $2n$. As there are Langford sequences on $n-1$ pairs for $n-1 \equiv -1$ or $0 \mod 4$, we see that Skolem can find his partitions for $n \equiv 0$ or $1 \mod 4$.

But Skolem wasn't finished. He added and subtracted 4 from the left-hand sides of (*), and obtained the equalities

$$12 - 11 = 1, 8 - 6 = 2, 10 - 7 = 3, \text{ and } 9 - 5 = 4.$$

This is a way to partition the numbers 1 through $3n$ into n triples $\{a, b, c\}$ such that $a+b = c$. For $n = 4$, we have the triples $\{1, 11, 12\}, \{2, 6, 8\}, \{3, 7, 10\},$ and $\{4, 5, 9\}$. Skolem used these solutions to find packings and coverings of certain graphs and to construct certain *combinatorial designs*. These are arrangements of into systems of subsets of a finite set that satisfy certain harmonious properties of balance and symmetry.

Skolem's sequences give us ways to partition finite sets of integers of size $2n$ into pairs whose differences are the numbers from 1 to n. But if we don't restrict ourselves to a finite set of integers, we can partition the set of all positive integers into two sequences $A = \{a_1, a_2, \ldots\}$ and $B = \{b_1, b_2, \ldots\}$ such that the differences $b_1 - a_1, b_2 - a_2, \ldots$ give us every positive integer without duplication. Figure 1 tells the story.

A and B are examples of *Beatty sequences*, which are sequences of the form $\{\lfloor n\alpha \rfloor : n = 1, 2, \ldots\}$, where α is an irrational number greater than 1 and $\lfloor x \rfloor$ denotes the greatest

A	**1**	**3**	4	6	**8**	9	11	12	14	16	17	19	**21**	...
B	**2**	**5**	7	10	**13**	15	18	20	23	26	28	31	**34**	...
difference	**1**	**2**	3	4	**5**	6	7	8	9	10	11	12	**13**	...

Figure 1: Beatty sequences.

integer in x. (**Query**: Notice anything interesting about the numbers in Figure 1 in **boldface**?)

In this example, $A = \{\lfloor n\phi \rfloor\}$ and $B = \{\lfloor n\phi^2 \rfloor\}$, where $\phi = (1+\sqrt{5})/2$ is the famous *golden section*; two such Beatty sequences are called *complementary.* Furthermore, ϕ is the positive root of the polynomial $x^2 - x - 1$; hence, $\phi^2 = \phi + 1$ and so dividing by ϕ^2 tells us that $1/\phi + 1\phi^2 = 1$. These two facts are at the heart of the following far-from-obvious theorem, named for Samuel Beatty, whose problem in the *American Mathematical Monthly* in 1926 called for a proof of the theorem.

BEATTY'S THEOREM. If α and β are irrational numbers greater than 1 that satisfy $1/\alpha + 1/\beta = 1$, then the sequences $\{\lfloor n\alpha \rfloor\}$ and $\{\lfloor n\beta \rfloor\}$ contain every positive integer without duplication.

But there is more. $\phi^2 = 1 + \phi$ implies $n\phi^2 = n + n\phi$ for $n = 1, 2, \ldots$ and so $\lfloor n\phi^2 \rfloor = \lfloor n + n\phi \rfloor = n + \lfloor \phi \rfloor$. It follows that $\lfloor n\phi^2 \rfloor - n\lfloor \phi \rfloor = n$ for all n, and so their *differences* give us every positive integer without duplication. (**Query**: are there two Beatty sequences whose differences give us every *even* integer without duplication? every multiple of 3?)

This bears repeating: not only are the two Beatty sequences A and B complementary, but the sequence of their differences is the set of positive integers in order.

There is a fourth connection, and that has to do with Wythoff's game, dating from 1905. In this game, two persons – Andrew and Betty – play a game with two piles of chips of sizes n and k. Let's call this the (n, k) position. The players take turns removing any number of chips from one pile or equal numbers from two piles, and the winner is the player who takes the last chip. If it's Andrew's turn and either one pile of size n or two equal piles of size k remain, this is a *winning position*: he wins by removing the lone pile or the two equal piles. What if it's Andrew's turn and the position is $(1, 2)$?

Well, if he takes the pile of 1, the pile of 2, or 1 from each pile, Betty is in the position $(0, 2), (1, 0,)$ or $(1, 1)$ – a single pile or two equal piles, and so Betty has a winning move. Thus, $(1, 2)$ is a *losing position*: no matter what move Andrew makes, Betty can win. The same thing happens with $(2, 1)$. Similarly, $(3, 5)$ is a losing position, because for each move Andrew can make, Betty can win directly or move to either $(1, 2)$ or $(2, 1)$. From a winning position, there is a move to a losing position, and from a losing position, no matter what move is made, the opponent can find a move to a losing position. Hmmm ... where have we seen $(1, 2)$ and $(3, 5)$ recently?

Figure 1 shows that if you play Wythoff's game so that you land on a position (n, k), where n and k are corresponding members of the Beatty sequences A and B, then you can win – and that is the Beatty-Wythoff connection.

(**Query**: If Beatty's problem was published 21 years after Wythoff's paper, why are the sequences named for Beatty?)

We have now seen the threads that weave the works of Langford and Skolem and Beatty and Wythoff together into part of a beautiful tapestry. The threads were always there, but Richard Guy was perceptive enough to do the weaving.

We'll get back to more of Richard's interesting math, but first, let's talk about Richard's coauthor, Bud Brown. That would be me.

5 Ezra (Bud) Brown, Mathematician and Writer

Ezra (Bud) Brown grew up in New Orleans in a family of teachers and writers. has degrees from Rice and LSU, taught at Virginia Tech for 48 years, and recently retired as Alumni Distinguished Professor Emeritus of Mathematics. He has done research in number theory, combinatorics, and expository mathematics – but one of his favorite papers is one he wrote with a sociologist. He is a frequent contributor to the MAA journals, and has been known to impersonate Alex Trebek at the spring meetings of the MD-DC-VA section of the MAA. Other pursuits include baking biscuits (they're better if you use softened butter), singing (anything from opera to rock'n'roll), playing jazz piano, and watching an occasional bird.

Enough about me. Let's get back to Richard's mathematics.

6 The Strong Law of Small Numbers

In 1988, Richard published "The Strong Law of Small Numbers" [6], a collection of thirty-five examples of patterns that seem to appear when we examine small values of n but for which, as Richard put it, "You can't tell by looking, because there aren't enough small numbers to meet the many demands made of them." Some of these patterns persist, some are figments of the small values of n, and for some, the answer is not known. Here are some of those examples:

1. Always primes ... or not?

   If $M_n := 2^n - 1$ is a prime, then n must be a prime, and $2^2 - 1 = 3, 2^3 - 1 = 7, 2^5 - 1 = 31, 2^7 - 1 = 127$ are all primes.

2. Always producing squares and cubes ... or not?

   Write down the positive integers, delete every second (boldfaced) number, and form the partial sums of those remaining:

   $$\begin{array}{ccccccccccccc} 1 & \mathbf{2} & 3 & \mathbf{4} & 5 & \mathbf{6} & 7 & \mathbf{8} & 9 & \mathbf{10} & 11 & \mathbf{12} & 13\ldots \\ 1 & & 4 & & 9 & & 16 & & 25 & & 36 & & 49\ldots \end{array}$$

   As above, but delete every third (boldfaced) number, then delete every second (boldfaced) partial sum:

1	2	**3**	4	5	**6**	7	8	**9**	10	11	**12**	13	14	**15**	16
1	**3**		7	**12**		19	**27**		37	**48**		61	**75**		91
1			8			27			64			125			216

3. Partitioning Circles — does the pattern persist?

   Place n points on a circle in such a way that no three of the chords joining them meet at a point. For $n = 1, 2, 3, 4,$ and 5, the number of regions into which the chords partition each circle equals 1, 2, 4, 8, and 16, respectively. See Figure 2.

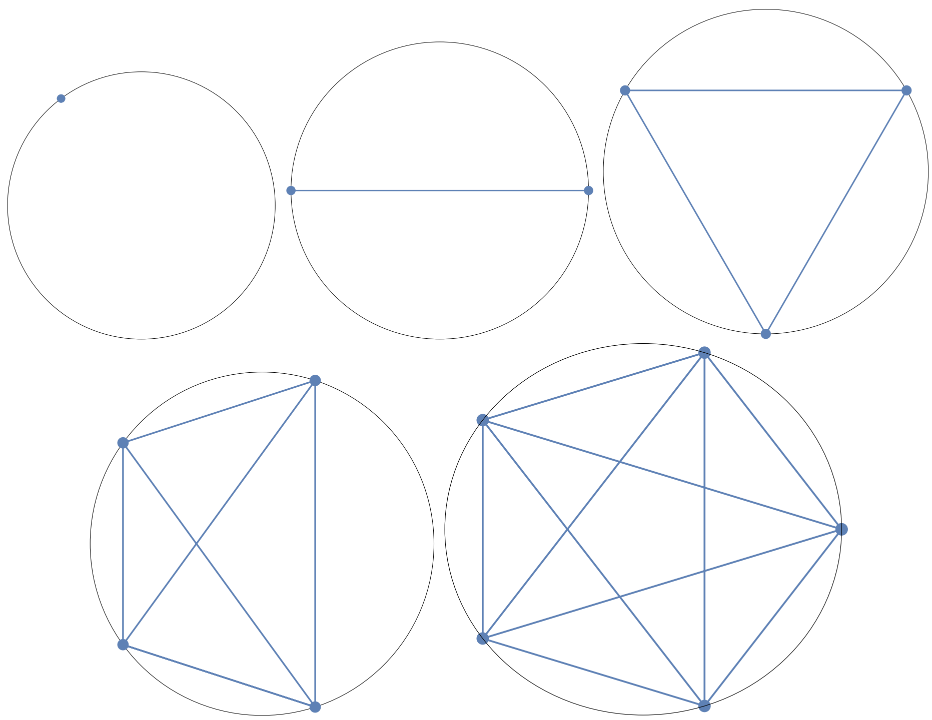

Figure 2: Partitioning a circle into regions.

The answers are in Section 9 . . . and now, some more history.

7 Brown Meets Guy

"I need expository papers for the *College Mathematics Journal (CMJ)*'s audience: college students and their teachers, especially in the first two years of the college curriculum. You can do this. Why not send me a few? Gotta run." And like the Red Queen, Woody Dudley – the *CMJ*'s new editor – disappeared.

Why not, indeed? So I sent three papers to Woody, one about square roots and two more about elliptic curves, and they all appeared in the *CMJ*.

My next paper, "The Many Names of $(7, 3, 1)$" [2], was about a combinatorial design – the drawing on the right in Figure 3 – that was also a round-robin tournament, an error-correcting code, and seven mutually adjacent hexagons drawn on a torus. I submitted the paper to *Mathematics Magazine* — like the *CMJ*, another journal for expository mathematics — and this led to my first connection with Richard.

The editor accepted the paper and sent a referee's report, which said that the paper was fine as written "... but here are a few suggestions that might improve the paper." The report concluded, "I don't care if you mention my name." The suggestions did improve the paper, so much so that it was selected for a writing award by the Mathematical Association of America – the MAA. That referee was none other than Richard Guy, and he was in the audience at MathFest 2003 when the paper was honored.

This was the beginning of my friendship with this gentlemanly giant of the world of numbers and sequences and patterns and games. Over the next seventeen years, we met at meetings and went to talks together – always sitting on the front row. He signed my copy of his book *Winning Ways for your Mathematical Plays*, adding these words: "To Bud Brown, a cheerful mathematician."

Richard's sense of humor was always present. For example, at MathFest 2006 Art Benjamin and I organized an invited paper session called "Gems of Recreational Mathematics" and Richard gave one of the four invited talks. He told Art and me that his colleagues at Calgary were organizing a symposium in honor of his upcoming 90th birthday, " ... but if I don't make it to 90, they'll turn it into a memorial!"

Shortly after MathFest 2003, he sent me a reprint (yes, a hard copy in the mail) of the *TUoC* paper. Little did I know how well I would get to know that 30-page document!

8 "I hereby pronounce you coauthors."

In the 1980s, Don Albers, the former MAA Director of Publications, went to Calgary to help Richard finish the delightful collection "The Lighter Side of Mathematics," at which time Don learned that Richard could probably hike anybody in the world into the ground, up to and including Sir Edmond Hillary. For many years after the *TUoC* paper appeared, Richard had talked with Don about expanding the paper into a Carus Monograph for the MAA. At one point, Don realized that Richard was not getting any younger, and his concern about *TUoC* seeing the light of print in book form was growing with each passing year. Let's let Don describe what happened next:

> "I knew that Richard liked Bud Brown's writing on number theory and combinatorics, and that Richard greatly enjoyed Bud's musical performances at banquets and his sense of humor. I also was aware that Bud wrote like a ball of fire. The three of us happened to be standing together in one of the halls at the Hartford MathFest in 2013 when the clouds parted, and the solution

> appeared like a lightning flash. The answer to my problem was to appoint Bud and Richard co-authors of *TUoC*, so I did on the spot. They shook hands, and the deal was done. The rest is history. I have had many joys in finding authors for MAA books; assembling the Brown-Guy team gave me a real glow." [1]

So, let's talk about writing the book. But first, the answers to those three "Are They Or Aren't They" questions.

9 The answers to the questions in Section 6

1. Always primes ... not. The numbers M_n are the Mersenne numbers, and although M_n is prime for $n = 2, 3, 5,$ and 7, $2^{11} - 1 = 2047 = 23 \cdot 89$ is composite. In the list of Mersenne numbers, primes are rare: as of 2019, only 51 of these numbers have proved to be primes, the largest being $2^{282,589,933} - 1$, a number of some eighty-five billion digits. The history of the Mersenne numbers is long and colorful: look it up.

2. Always producing squares and cubes: true. The "strike it out — add it up" pattern, conjectured by Alfred Moessner in a 1951 paper (in German), does indeed produce squares, cubes, and higher powers, as well as factorials. Kozen and Silva's *Monthly* paper[7] has a proof of Moessner's original conjecture as well as extensions.

3. The pattern $1, 2, 4, 8, 16, \ldots$ persists ... false! For six points on a circle, the number R of regions is 31, not 32 – which certainly amazed me when I first saw it. For n points on a circle, the number of regions R is given by the formula

$$R = \binom{n}{4} + \binom{n}{2} + 1 = \binom{n-1}{4} + \binom{n-1}{3} + \binom{n-1}{2} + \binom{n-1}{1} + \binom{n-1}{0}.$$

   Richard gives a proof in [6] that uses Euler's formula $V - E + R = 2$ for planar graphs, the partitioned circles being graphs in the plane with $V = n + \binom{n}{4}$ and $E = 2\binom{n}{4} + \binom{n+1}{2}$. Euler's formula includes the region outside the circle, so there are $E - V + 1$ finite regions.

Now, back to our story.

10 Working with Richard: *TUoC* becomes a book

For a few months after the Hartford meeting, we collected ideas. Then, in February 2014, Richard sent an email outlining his concept of the book with a chapter-by-chapter list of topics to include. Together with additional material in support of the original topics, that outline was the blueprint for the book, and we set to work.

Eventually, we got into a rhythm. I would write up a section or part of a chapter and send it to Richard and to our two editors, Fernando Gouvea and Steve Kennedy. Some of the topics, especially combinatorial designs, were familiar to me. Others were not. If my knowledge of a topic was scanty, I studied the topic until I could explain that topic to students and mathematicians and others who knew even less than I did. This I did with Beatty sequences, the Penrose tilings and Conway worms, the coin-turning games of Chapter 15, and the Steiner system $S(5, 8, 24)$.

If I got to something I simply did not understand, I took it out. There only two of those somethings, thankfully. Certain of Richard's original topics did not make the final cut because it would have required too much background to allow the reader to see combinatorial connections. For example, Richard's section on factoring integers with quadratic forms was omitted because it would have required too much background in number theory, and we were not writing a number theory book.

For the 30-page Chapter 8 on geometric connections in combinatorics, our editor Fernando Gouvea suggested that Section 8.1 should include a rigorous exposition of the fundamentals of projective geometry. Since Fernando is a geometer, I had to do it right, which meant having a deep understanding of what I was writing about. The book grew and expanded over the years and we agreed that any additions should show plenty of connections to topics already in the book.

Finally, the drawings in Figure 3 are two figures whose many connections and threads epitomize what the book is all about: Rick's Tricky Six Puzzle (RT6) [4] is a sliding

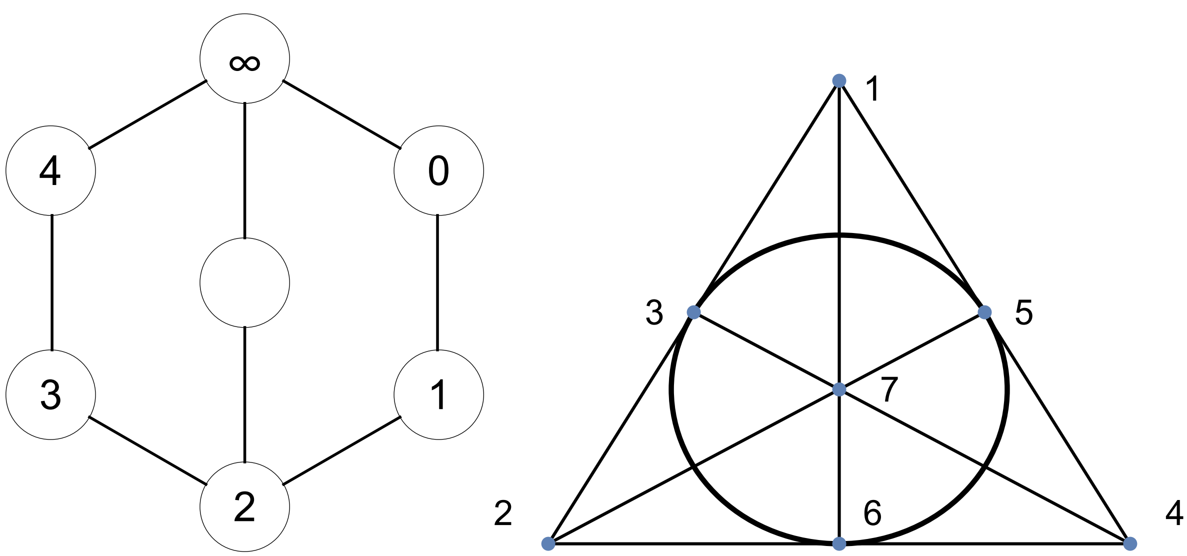

Figure 3: Rick's Tricky Six Puzzle (left) and the $(7, 3, 1)$ block design (right)

block puzzle similar to Sam Loyd's famous Fifteen Puzzle. In all sliding block puzzles – with a single exception – either every position can be reached from every other position, or there are two disjoint sets of mutually reachable positions, with positions in one set unreachable from positions in the other set. The single exception is the RT6 puzzle with **six** pairwise unreachable sets of positions. RT6 has connections with some two dozen other combinatorial objects, and Bud was a referee for the RT6 paper. Finally, the $(7,3,1)$ block design is similarly connected to at least twenty-one other combinatorial objects and as we have heard, Richard was a referee for Bud's "The many names of $(7,3,1)$."

And you know what? We finished the manuscript and submitted it to Jennifer Sharp at the American Mathematical Society on Halloween, 31 OCT, 2019 — (**Query**: Why is 31 OCT = 25 DEC?) — and *TUoC*: The Book was published in May 2020.

I regret to say that Richard did not live to see the book published. Our last communication was about the cover design, and we agreed that it was the best choice.

Richard Kenneth Guy died March 9 2020. He was one hundred and three years young. Nobody can fill the gap he left.

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[4] Alex Fink and Richard K. Guy, Rick's Tricky Six Puzzle: S_5 sits specially in S_6, *Math. Magazine* **82** (2009), pp. 84–102.

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[7] Dexter Kozen and Alexandra Silva, On Moessner's Theorem, *Amer. Math.Monthly* **120** (2013), pp. 131-139.

Ezra (Bud) Brown retired from Virginia Tech in 2017 as Alumni Distinguished Professor Emeritus of Mathematics. If you want to know more about him, read the preceding article.